

Fractal Geometry

{ Shannon Keenan

- ⌘ <http://www.youtube.com/watch?v=AfHvtmLC1Y>
- ⌘ Endlessly repeating geometric figures on a diminishing scale (self-similarity)
- ⌘ Infinitely complex
- ⌘ Picture that tells the story of the mathematical process that produces it

What is a Fractal?



- ⌘ **Gottfried Leibniz**- considered recursive self-similarity
- ⌘ **Karl Weierstrass**- continuous everywhere, differentiable nowhere
- ⌘ **Georg Cantor**- subsets of the real line
- ⌘ **Helge von Koch**- Koch Curve
- ⌘ **Gaston Julia**- mapping of complex numbers

Some History

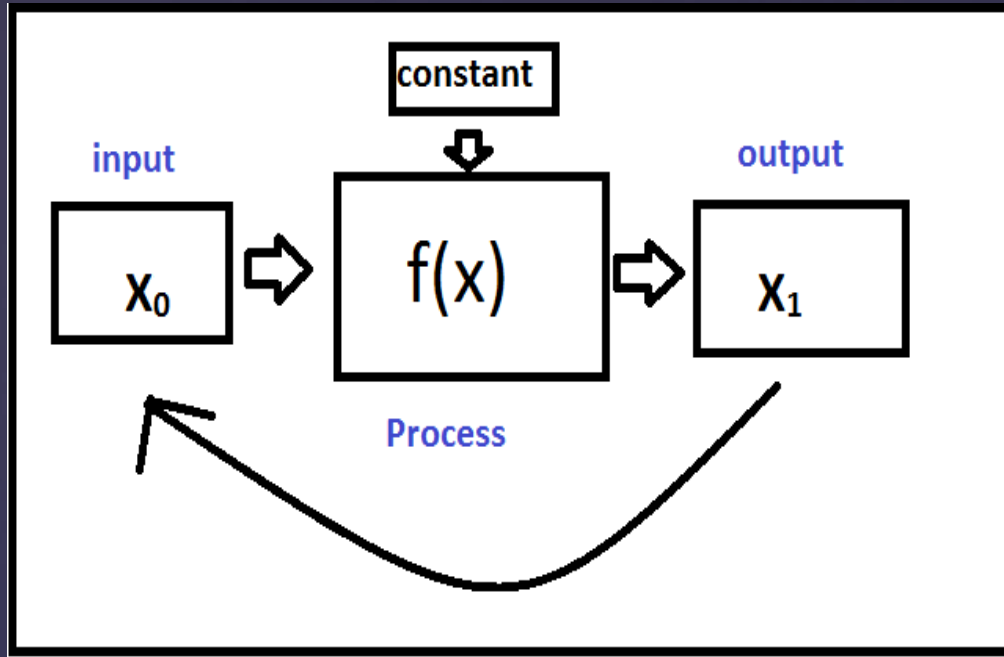
{ ideas

- ⌘ Born in Poland 1924, educated in France
- ⌘ Gave fractals a formal definition
- ⌘ *"Mountains are not cones, clouds are not spheres, bark is not smooth, nor does lightning travel in a straight line"*
- ⌘ Definition came with the invention of the computer
- ⌘ *"The Fractal Geometry of Nature"*

{ definition

- ⌘ Complex Self-similarity
- ⌘ Fractal dimension
- ⌘ Never an exact definition

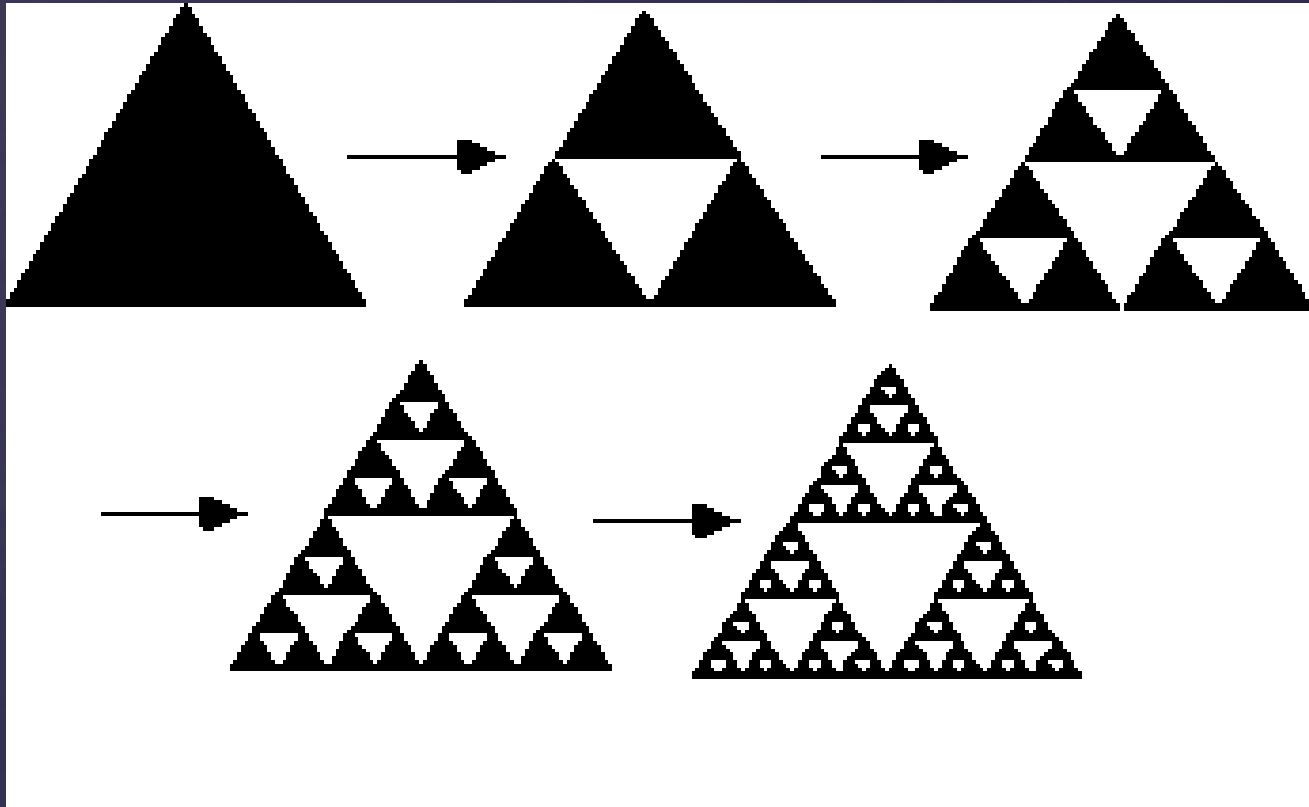
Benoit Mandelbrot



- & The output of one process is the input of the next
- & We see this a lot in calculus
- & Also can be two step (Fibonacci)
- & or chance (chaos)
- & Transformation can be nonlinear

Feedback Loop

the process that creates self-similarity

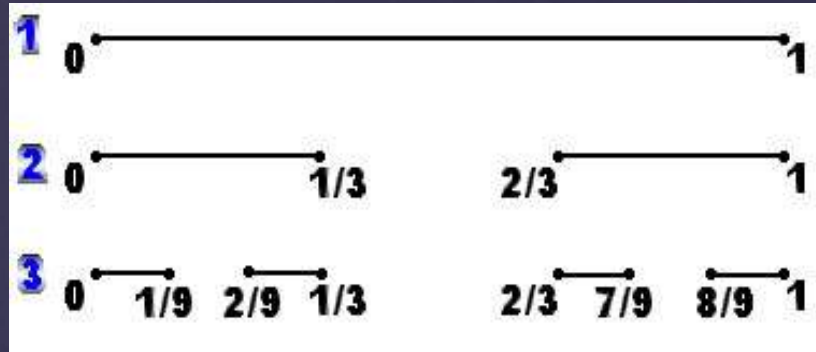


Sierpinski Triangle

- ⌘ We know zero to three dimensions
- ⌘ Derive dimension definition
- ⌘ $n=3^d$
- ⌘ $D = \log(n)/\log(3)$

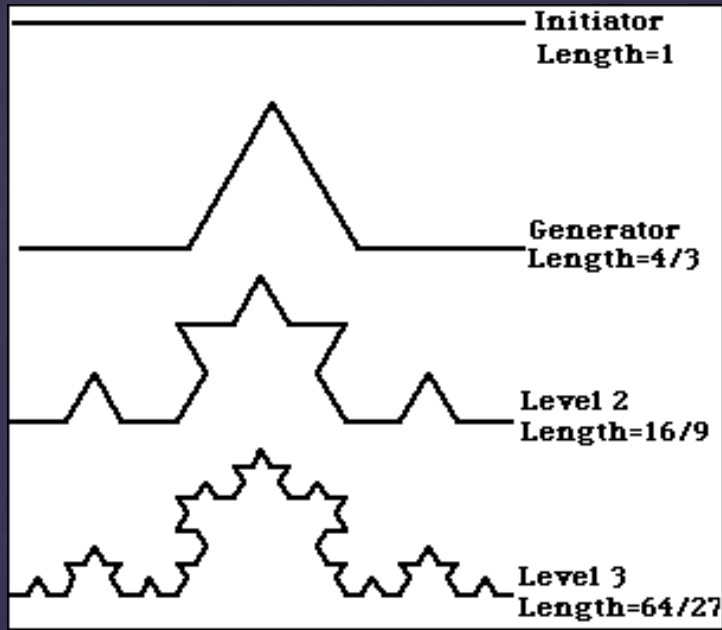
Object	Number of Copies	Formula	Dimension
Line	3	3^1	1
Square	9	3^2	2
Cube	27	3^3	3

Fractal Dimension



- Length Removed Becomes equal to 1
- But an infinite number of points
- What is the dimension?
- $D = \log(2)/\log(3) = 0.63\dots$

Cantor Set

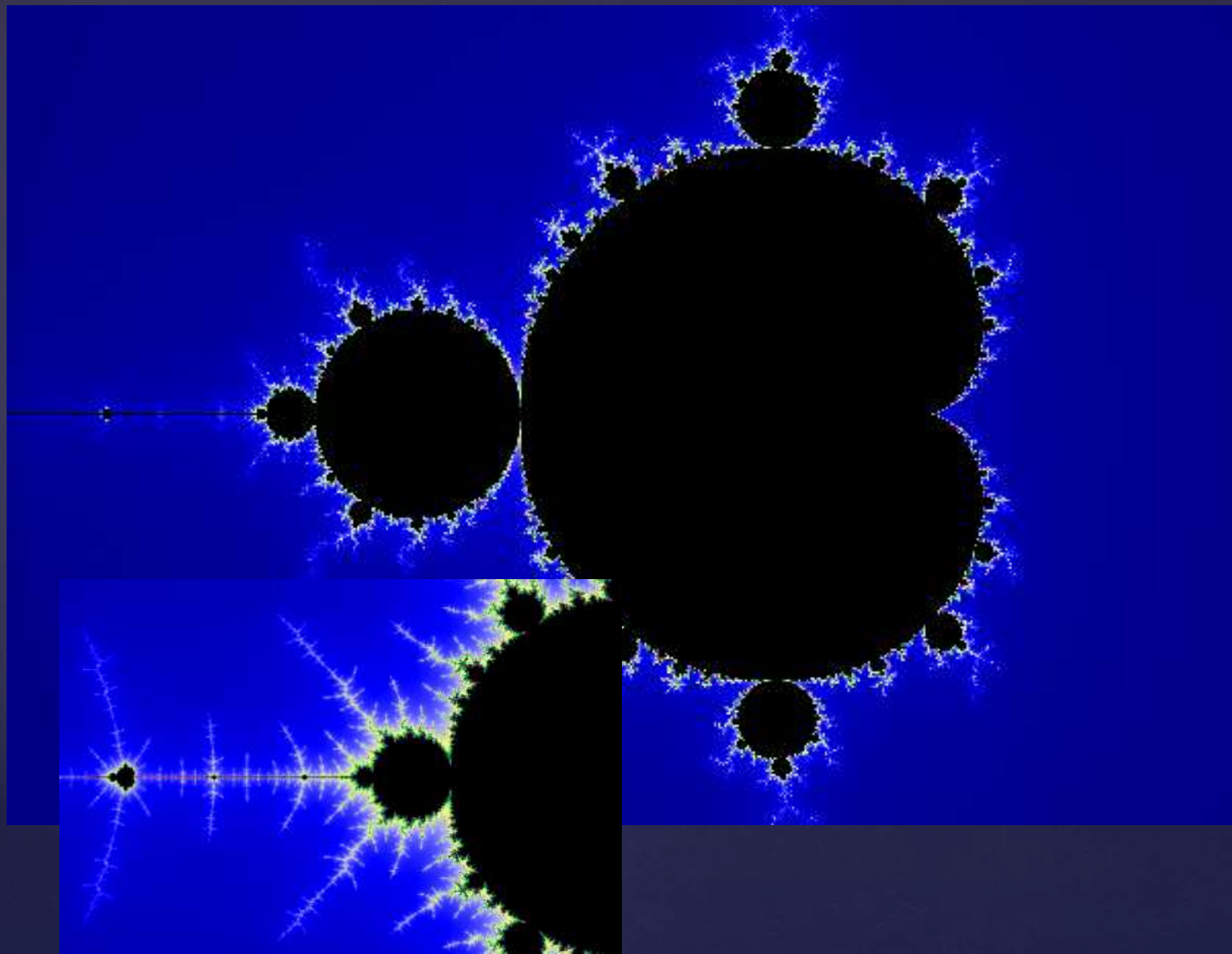


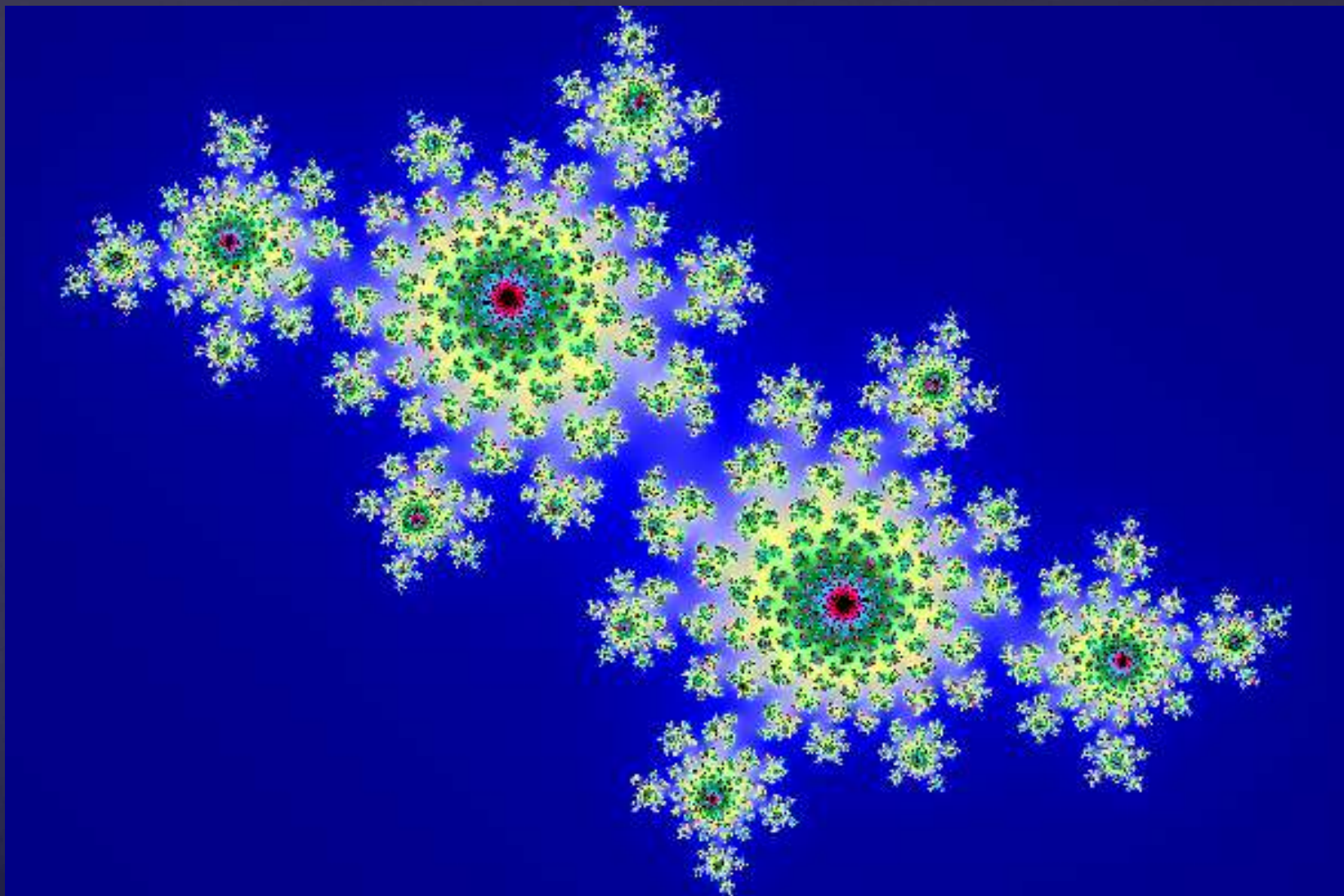
- Similar to a natural coastline
- Continuous everywhere, differentiable nowhere
- How long is the Koch Curve?

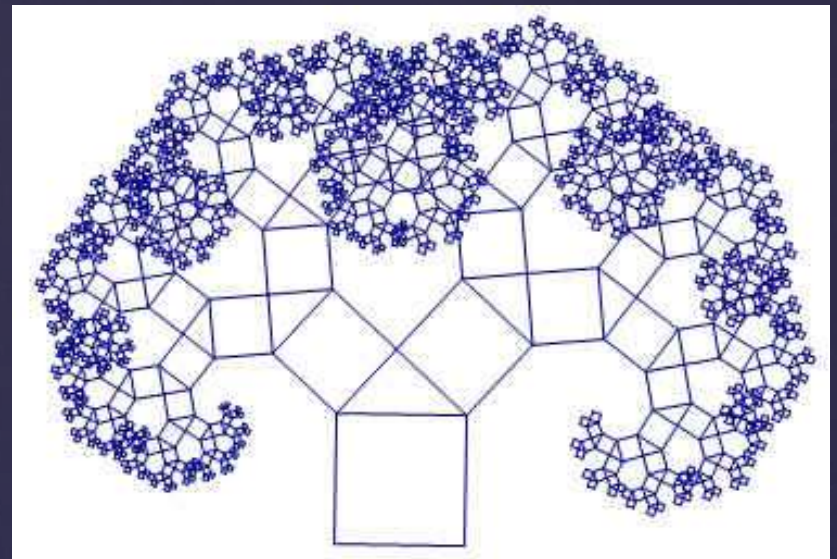
Other Interesting Fractals: Koch Curve

- ⌘ Live on the Complex plane
- ⌘ Useful images for explaining what the function looks like
- ⌘ “If f is an entire function- if particular, when f is a polynomial, then $J(f)$ is the boundary of the set of points which diverge to infinity under iteration”
- ⌘ $x \rightarrow x^2 + c \rightarrow (x^2 + c)^2 + c \rightarrow ((x^2 + c)^2 + c)^2 + c \dots$

Julia Fractal







- & http://scienceblogs.com/chaoticutopia/2006/11/friday_fractal_xxv.php
- & <http://library.thinkquest.org/2647/chaos/cantor.htm>
- & [http://www.castledragmire.com/Projects/Julia Fractals](http://www.castledragmire.com/Projects/Julia_Fractals)
- & <http://web.me.com/paulscott.info/fractals-in-nature/1/S7.ammonite-fossil.html>

Resources