Orthogonal Trajectories

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What is it?!

- An Orthogonal Trajectory is:

- **An application of differential equations**
- A curve that intersects each curve of a family of functions at right angles
- Every point of intersection is a right angle!
- Orthogonal means right angle
- Trajectory refers to a curve
- Used in physics and engineering

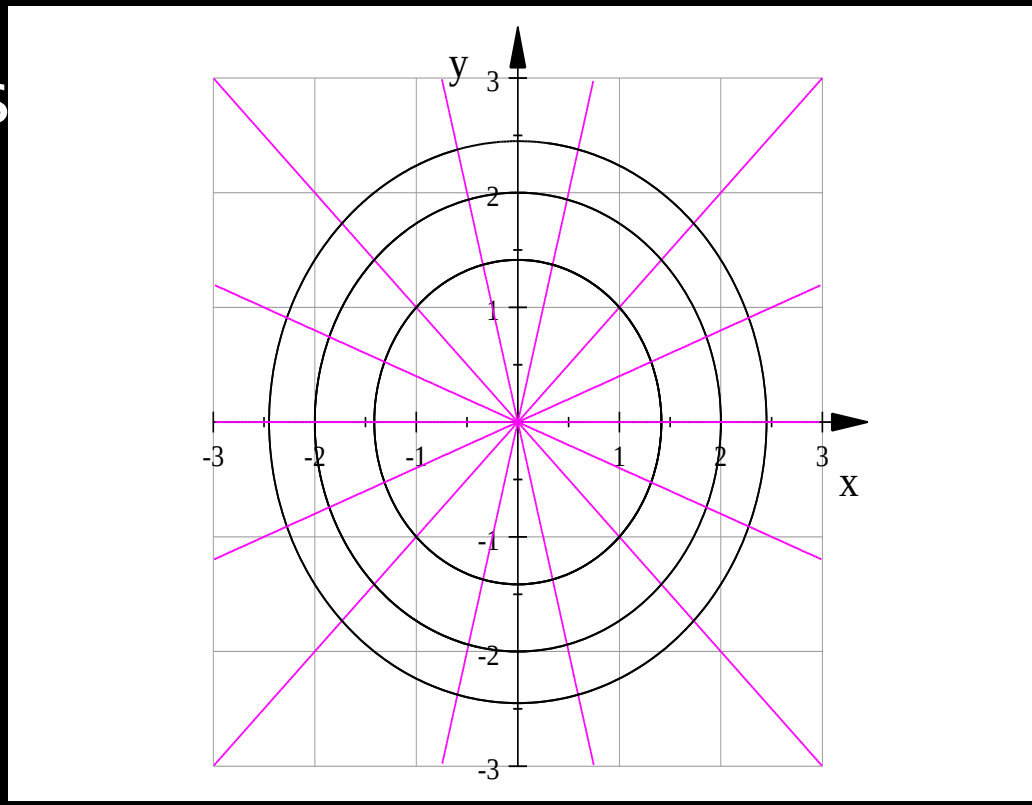
Tell me how it's done!

- Remember that the slopes of perpendicular lines are negative reciprocals of each other

- The following steps are a general guideline for how to solve for a an orthogonal trajectory:
- Find a single differential equation that is satisfied by every member of a family
- This is the slope of the tangent line at any point for any member of the family
- For an orthogonal trajectory, the slope of the tangent line is the negative reciprocal of this slope
- Solve this new differential equation
- This is the solution!

See the graph

- Take the family $y=mx$, straight lines which run through the origin
- It is an orthogonal trajectory of $x^2+y^2=r^2$ –concentric circles with a center at the origin



Do the Math

- Take the family: $x^2+y^2=r^2$
- Find a differential equation that satisfies every member of the family
- Differentiate implicitly: $2x+2y(dy/dx)=0$
- $2y(dy/dx)=-2x$
- $dy/dx= -2x/2y$
- $Dy/dx=-x/y$ is the slope of the tangent line on any point on a circle with a center at the origin

- $Dy/dx=-x/y$
- An orthogonal trajectory's tangent line has a slope which is the negative reciprocal of this slope:
- $Dy/dx= y/x$
- You now have a separable differential equation
- Separate and integrate the linear separable differential equation:
- $C(x)=e^{-\int 1/x\,dx} = 1/x$ use an integration factor
- $Y=m/c(x)$
- $y=mx$
- So, the family $y=mx$ is an orthogonal trajectory of the family $x^2+y^2=r^2$

One more example

- Family: $y=k/x$ see that $k=xy$
- $dy/dx= -k/x^2$
- $Dy/dx =-xy/x^2$
- Slope of the tangent of the orthogonal trajectory $dy/dx= x/y$
- Separate and integrate:
- $\int y\,dy=\int x\,dx$
- $1/2y^2=1/2x^2+k$
- $y^2=x^2+k$
- $K= y^2-x^2$
- $K= y^2-x^2$ is an orthogonal trajectory to the family $y=k/x$