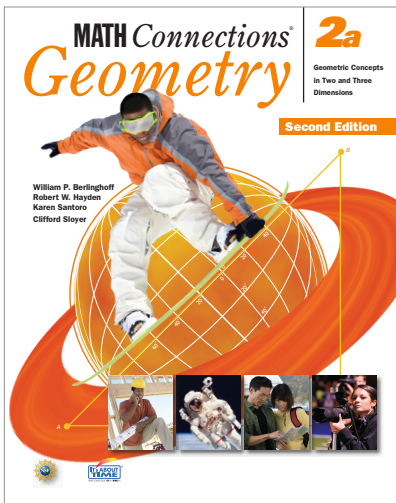


MATH PROGRAMS

Geometry Math Lessons for Middle and High School



The Building Blocks of Geometry: Making and Measuring Polygons



1.1 Measuring Lengths

Learning Outcomes
After studying this section, you will be able to:
• Explain what a mathematical model is.
• Describe differences between a model and a real-world model.
• Set the difference between discrete and continuous quantities.
• Describe when a unit of measurement and a scale are used.
• Measure a given length with a given unit.

You can think of geometry as the visual side of mathematics. It deals with shape, form, and size. We begin this book by looking carefully at some of the surprisingly simple ideas on which all of geometry rests. Really understanding these few simple ideas is the key to this powerful part of mathematics.

Earlier in MATH Connections, you used geometry to visualize some other things you were doing. For example, you used coordinate geometry to make graphs of functions and equations. You also used your diagrams to visualize counting processes. One role of geometry is to provide pictures that help understand some other part of mathematics. A picture often gives you a quick grasp of concepts that can be made more precise with symbols and calculations. The connection between geometric figures and algebra is arithmetic in measurement. When we measure a geometric shape, we get numbers. We can do arithmetic with the numbers or plug them into an algebraic equation. For this reason, we start with measurement.

Introduction to Trigonometry: Tangles With Angles

1.1 Using your calculator's SIN function, find the angle θ (Justify your answer).

7.31
Display 3.37

2. Now calculate SIN θ directly from your answer to part 1. How is this number related to 7.31?

3. Using your calculator's COS⁻¹ function, find the angle α in Display 3.38 to the nearest hundredth of a degree.

8.24
5.73
Display 3.38

4. Now calculate COS⁻¹ directly from your answer to part 1. How is this number related to 7.31?

We have defined the functions sine, cosine, and tangent as ratios of sides of a right triangle. These definitions work for acute angles. That is, each of these functions makes an angle θ greater than 0° and less than 90° with a number. Later in this chapter, you will see how to extend these definitions to other angles, but for now we shall think of the domain of these three functions as $\{0^\circ < \theta < 90^\circ\}$.

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3.4 Going Backwards: The Inverse Trig Functions

1. What is the range of \sin^{-1} of \cos^{-1} of \tan^{-1} ? Justify your answers.

2. \sin^{-1} , \cos^{-1} , and \tan^{-1} are functions, too, so each one must have a domain and a range. What is the domain of \sin^{-1} of \cos^{-1} of \tan^{-1} ? Justify your answers.

3. What is the range of \sin^{-1} of \cos^{-1} of \tan^{-1} ? Justify your answers.

Answer these questions without using your calculator. Be prepared to justify your answers.

1. $\tan^{-1} 1 = \dots$

2. $\cos^{-1}(\cos 35^\circ) = \dots$

3. $\sin^{-1}(\sin 6.75^\circ) = \dots$

Here is an example of a situation in which an inverse function is useful. The space station ISS is put in an equatorial orbit 500 miles above Earth. Tracking stations are to be placed along the equator to monitor it. Each tracking station has a scanner that covers 180° with the horizon, as shown in Display 3.39.

The problem is to place tracking stations close enough to each other so that the ISS can always be picked up by at least one scanner. If the tracking stations are too far apart, there will be gaps in ISS's orbit where none of the scanners can reach it as shown in Display 3.40.

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Tracking stations are expensive to build and maintain so we want to have as few of them as possible. Thus, we want to place the length of the arc (part of the circumference) d as in Display 3.42.

Using 4000 miles as Earth's radius and observing the length in Display 3.41, you can see that if we find the length of the arc (part of the circumference) $d = 7.842$.

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Introduction to Trigonometry: Tangles With Angles

3.4 Going Backwards: The Inverse Trig Functions

Problem Set 3.4

1. The calculator will give you practice in using your calculator's inverse trig functions. For each figure, find the measure of angle θ to the nearest hundredth of a degree.

2. A rectangle is 24 centimeters long and 10 centimeters wide. Find the angles that a diagonal makes with the sides.

3. A 40-foot ladder is used to reach the top of a 10-foot wall. If the ladder extends 4 feet past the top of the wall, how angle does the ladder form with the horizontal ground?

4. A space station is in an equatorial orbit 200 miles above Earth. You want to place tracking stations along the equator. Each tracking station has a scanner which covers 180° with the horizon. What is the greatest distance allowed between two tracking stations so that there are no gaps?

5. A 20-foot vertical flagpole casts a $\frac{1}{2}$ -foot shadow onto the flat ground.

(a) How many degrees is the sun below the direct overhead direction? Round to the nearest tenth of a degree.

(b) If the horizontal shadow of a nearby tree is 7.8 feet long, how tall is the tree?

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Introduction to Trigonometry: Tangles With Angles

Looking Back

This chapter has been about trigonometry, the study of connections between sides and angles of triangles. Trigonometric functions are powerful tools for solving measurement problems without making scaled drawings.

First you learned how the three basic trigonometric functions—sine, cosine, and tangent—defined their acute angles. They are ratios of particular sides lengths from any right triangle that contains the angle. The reciprocals of these are the functions cotangent, secant, and cosecant. The inverse functions are \sin^{-1} , \cos^{-1} , and \tan^{-1} , relate the ratio values back to their angle values.

You used trigonometry with vectors to solve problems about directed forces, such as wind. Vectors can be described by magnitude and direction or by rectangular coordinates. The sine and cosine functions for you convert magnitude and direction into coordinates. The \tan^{-1} function lets you convert coordinates into direction, in degrees with respect to the x -axis.

Trigonometric identities are laws that are true for any angle. An important example is the trigonometric version of the Pythagorean Theorem, $\sin^2 \theta + \cos^2 \theta = 1$. Two others are the Law of Sines and the Law of Cosines. You used them with triangulation and triangle congruence tests to calculate lengths and areas for some very irregular polygons!

Finally, you learned how sine and cosine can be defined for any angle. As a point moves around a unit circle centered at the origin, the ratios for that point makes an angle θ with the positive x -axis. Then $\sin \theta$ is the point's y -coordinate, and $\cos \theta$ is its x -coordinate. The graphs of these functions form a "wave" pattern that repeats as the angle rotates around and around the circle. This pattern, often called a sine wave, occurs in many areas of science and technology. You will learn more about these graphs in Year 3 of MATH Connections.

In this chapter you learned how to:

- Describe the six basic trigonometric functions of an acute angle θ as a right triangle as ratios of lengths.
- Use $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$, $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$, and $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$.
- Use \sin^{-1} , \cos^{-1} , and \tan^{-1} to find an angle θ from a ratio.
- Use the Law of Sines and the Law of Cosines to solve measurement problems.
- Relate the coordinates of points on the unit circle to the sine and cosine of angles.
- Extend the domains of the functions \sin , \cos , and \tan .
- Explain the shape of the graphs of $y = \sin x$ and $y = \cos x$.
- Describe an equation for a circle in both algebraic and trigonometric forms.

Along the way, you also learned to:

- Describe vectors by magnitude and direction and by rectangular coordinates.
- Add and subtract two vectors.
- Multiply a vector by a scalar.
- Use the vectors to solve problems about directed forces.

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Introduction to Trigonometry: Tangles With Angles

Looking Back

Describe how sine is related to cosine and how cosecant, secant, and cotangent are related to sine, cosine, and tangent; and State and explain the trigonometric form of the Pythagorean Theorem.

$\sin^2 \theta + \cos^2 \theta = 1$

Explain how \sin^{-1} , \cos^{-1} , and \tan^{-1} can be viewed as inverse functions, and identify suitable domains and ranges for them.

Use the calculator to find trigonometric and inverse trigonometric values.

Use the trigonometric functions and their inverses as problem-solving tools.

Find rectangular coordinates of a vector using sine and cosine.

Explain the Law of Sines. For any triangle ABC , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Explain the Law of Cosines. In any triangle, for any angle θ with opposite side c , $c^2 = a^2 + b^2 - 2ab \cos \theta$.

Relate the coordinates of points on the unit circle to the sine and cosine of angles.

Extend the domains of the functions \sin , \cos , and \tan .

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Introduction to Trigonometry: Tangles With Angles

Review Exercises

Round all of your answers to two decimal places.

1. A guy wire is to be attached to the top of a 90-foot radio tower, as shown in Display 3.76. The angle θ is the degree measure of the angle between the wire and the vertical line through the top of the tower, and L is the length in feet of the guy wire.

(a) The engineers have decided to make $\theta = 20^\circ$. Find L .

(b) The engineers have decided to make the guy wire 100 feet long ($L = 100$). What will be the degree measure of the angle θ ?

2. Sasha wants to build a ramp from her lawn up to her deck to make it easier to move her grill. The deck is 75 centimeters off the ground, and she wants the ramp to be at an angle of 30° with the ground.

(a) Draw and label a diagram to help Sasha design the ramp.

(b) What will be the length of the ramp in centimeters? Is it correct?

(c) Sasha realizes that the ramp would be easier to use if it started at a walkway that is 200 centimeters from the deck. What would the ramp angle with the ground be then? What would the ramp length be in meters?

3. A sign says that a road has a grade of 7% percent, which means that it rises vertically 7 feet for every 100 feet of horizontal distance.

(a) Draw and label a diagram of this situation.

(b) What is the degree measure of the angle of incline? (c) What is the grade of a road that has a 9° angle of incline? **4.** Dominique has a 20-foot ladder. He places it against the side of his house so that it is the degree measure of the angle between the ladder and the ground, and d is the distance from the base of the ladder to the base of the house at this situation.

(a) Draw and label a diagram of this situation.

(b) Find d if $\theta = 58^\circ$.

(c) Find d if $\theta = 66^\circ$.

(d) Find d if $\theta = 18$ feet.

(e) Find d if $d = 31$ feet.

5. Display 3.77 is a diagram of the side view of a house with some dimensions labeled.

(a) Find the degree measure of the angle that the left side of the roof makes with the attic floor.

(b) Find the degree measure of the angle θ that the right side of the roof makes with the attic floor.

6. Ozama made a lean-to addition onto two large plywood boards, as shown in Display 3.78. One board is 4 feet by 3 feet and the other is 4 feet by 1 foot. The angle that the 4-foot board makes with the ground is 52° .

(a) What is the degree measure of the angle θ that the 4-foot board makes with the ground?

(b) What is the angle at the peak of the lean-to?

(c) What is the width in feet, w , of the area on the ground covered by the lean-to?

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Introduction to Trigonometry: Tangles With Angles

Review Exercises

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