Mathematics Theories and applications

Summary

This report explores the evolution of mathematics from counting and number systems to complex theories. It covers the development of logic, Euclid's algorithm, cryptography, calculus, graph theory, topology, Fourier analysis, group theory, set theory, Markov chains, game theory, chaos theory, geodesics, and Fermat's Last Theorem. Each concept is briefly explained, highlighting its significance and applications in various fields.

Takeaways

- 🔢 The origin of mathematics is rooted in numbers and counting, with the base-10 system likely originating from humans having 10 fingers.
- 🤔 The philosophical question of whether numbers were invented or pre-existing is still debated.
- 📚 Binary system uses only two digits, and a hypothetical base-12 system could simplify arithmetic due to 12's divisibility.
- 🧠 Logic is a foundational aspect of mathematics, with implications in problem-solving and theorem proving.
- 📐 Euclid's Elements introduced concepts like the Euclidean algorithm, fundamental to computing the greatest common divisor.
- 🔐 Cryptography, which protects sensitive data, relies heavily on number theory and is employed by organizations like the NSA.
- 📉 The discovery of mathematical constants like Pi and the development of calculus have had profound impacts on various scientific fields.
- 🌐 Graph theory, emerging from Euler's work on the seven bridges of Königsberg, has broad applications in computer science and social network analysis.
- 🔍 Topology examines the properties of space that are preserved under continuous transformations, with applications in physics and biology.
- 🎛️ Fourier transform allows any function to be decomposed into sums of sines and cosines, with wide applications in signal processing and quantum mechanics.
- 🔗 Group theory, starting from the early 1800s, studies symmetry within mathematical structures and has applications in chemistry and physics.

GREATEST MATH THEORIES

Explained

ThoughtThrill
3 Jun 202409:18

Summary

TLDRThis script delves into various mathematical concepts, starting with the Pythagorean theorem for right-angled triangles, then explores probability, calculus, and Einstein's theory of relativity. It touches on game theory, chaos theory, and number theory, including prime numbers. The script also covers topology, set theory, graph theory, linear algebra, and complex numbers. It concludes with topics like fractal geometry, Boolean algebra, non-Euclidean geometry, logarithms, exponentials, ring theory, combinatorics, transfinite numbers, and cryptography, providing a comprehensive overview of mathematical theories and their applications.

Takeaways

- 📐 The Pythagorean theorem is a fundamental principle in geometry for right-angled triangles, stating that the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- 🎲 Probability theory quantifies the likelihood of events, with outcomes ranging from impossible (0%) to certain (100%), and is used to predict occurrences like coin flips or lottery numbers.
- 📈 Calculus, with its fundamental theorem, is the study of change, connecting differentiation (rate of change) and integration (accumulation), useful for determining speed and distance in motion.
- 🌌 Einstein's theory of relativity, encompassing both special and general relativity, describes the connection between space and time, and how they are affected by motion and gravity.
- 🎲 Game Theory examines strategic situations where players' decisions affect each other, predicting outcomes through analysis of strategies and the Nash equilibrium concept.
- 🌪 Chaos Theory explores how minor changes in initial conditions can lead to significant differences in outcomes, exemplified by the butterfly effect in weather prediction.
- 🔢 Number Theory focuses on the properties of numbers, especially prime numbers, which are integral to various mathematical disciplines.
- 🔍 Topology, characterized by Euler's characteristic, studies properties of shapes and spaces that remain under continuous deformations, aiding in understanding fundamental structures.
- 🤖 Boolean Algebra operates with true or false values and is fundamental to computer science and digital logic for performing logical operations.
- 🔄 The Fourier Transform is a mathematical technique that breaks down complex signals into their constituent frequencies, used extensively in signal processing.
- 🔢 Linear Algebra deals with linear equations and their representations through matrices and vector spaces, essential for solving systems of linear equations.

- 🧭 Complex Numbers extend the concept of real numbers to two dimensions, useful for representing locations and directions, akin to coordinates on a map.
- 🌿 Fractal Geometry investigates self-similar patterns at different scales, found in natural phenomena like tree branches and snowflakes.
- ⛓ Ring Theory studies rings, sets with addition and multiplication operations, generalizing the arithmetic of whole numbers.
- 🎩 Combinatorics is the mathematical study of counting, arranging, and combining objects, addressing questions of arrangement and combination possibilities.
- ∞ Transfinite Numbers describe the sizes of infinitely large sets, extending the concept of counting numbers and representing different levels of infinity.
- 🔒 Cryptography is the practice of securing information through encoding messages, ensuring that only authorized parties can access the information.

Q & A

- **What does the Pythagorean theorem state about right-angled triangles?**

  -The Pythagorean theorem states that in a right-angled triangle with one 90° angle, the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the lengths of the other two sides (the legs).

- **How is the probability of an event measured?**

  -Probability measures the likelihood of an event occurring and is expressed as a number between zero (impossible) and one (certain). It helps predict the frequency of an event, such as the chance of rolling a six on a dice.

- **What are the two main concepts linked by the fundamental theorem of calculus?**

  -The fundamental theorem of calculus links differentiation, which measures how a quantity changes, and integration, which measures the total accumulation of a quantity, showing that these two ideas are connected.

- **How does the theory of relativity describe the relationship between time and space?**

  -The theory of relativity, proposed by Albert Einstein, shows that time and space are interconnected. It explains that time moves slower for objects moving faster or in stronger gravitational fields, and gravity bends space and time around massive objects.

- **What is the Nash equilibrium in Game Theory?**

  -The Nash equilibrium is a concept in Game Theory where no player can benefit by changing their strategy, given that all other players keep their strategies unchanged. It helps predict the outcome of decision-making situations.

- **What is the butterfly effect in Chaos Theory?**

  -The butterfly effect in Chaos Theory refers to the idea that small changes in the initial conditions of a system can lead to vastly different outcomes. It illustrates how seemingly insignificant events can have significant impacts.

- **What are prime numbers in number theory?**

  -Prime numbers are whole numbers greater than one that have no positive divisors other than one and themselves, such as 2, 3, 5, and 7.

- **How is Euler's characteristic used in topology?**

  -Euler's characteristic is a number that describes a topological space's shape or structure, calculated as V - E + F, where V is the number of vertices, E is the number of edges, and F is the number of faces.

- **What is the purpose of the Bayes' theorem in probability?**

  -Bayes' theorem helps update predictions or beliefs about the likelihood of an event happening given new evidence. It is used to calculate the probability of an event based on prior knowledge of related conditions.

- **What does Fermat's Last Theorem state about equations with powers greater than two?**

  -Fermat's Last Theorem states that there are no whole number solutions to the equation x^n + y^n = z^n for any integer value of n greater than two.

- **How does set theory help in understanding collections of objects?**

  -Set theory helps understand how collections of objects interact with each other, such as how they can overlap or be combined, by studying sets, which are collections of objects that can be numbers, letters, or even other sets.

Q & A

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81 Math Symbols Explained

Summary

TLDRThis script introduces fundamental mathematical symbols and their meanings, including basic arithmetic signs, equality and inequality, set theory symbols, logical operators, derivatives, integrals, and complex numbers. It also covers number systems, summation, limits, and cardinality of infinite sets, providing a comprehensive overview of mathematical notation and its applications.

Takeaways

- 🔢 The plus sign (+) is universally recognized for addition, while the minus sign (-) represents subtraction and denotes negative numbers.
- ✖️ The multiplication sign (*) is used for multiplying numbers and can also be represented as a dot (·).
- 🔄 Division is signified by the division sign (÷), which is sometimes written as a slash (/), and is the opposite of multiplication.
- 🔁 The plus-minus (±) and minus-plus (∓) signs are used to denote a range of values or the opposite sign, respectively.
- √ The root symbol denotes the square root, and with an integer superscript, it can represent the nth root of a number.
- ✅ The equals sign (=) is used to denote equality between two expressions, while the not-equal sign (≠) indicates inequality.
- ≈ The approximately equal sign (~) is used when two values are not exactly equal but are close, and also denotes similarity or proportionality.
- ≡ The triple bar or equivalence sign is used for identity or congruence in modular arithmetic.
- < & > The less than (<) and greater than (>) signs indicate the relative size of quantities, with their counterparts (≤, ≥) indicating equality or inequality as well.
- ∅ The empty set symbol denotes a set with no elements, while the number sign (#) often denotes the cardinality of a set.
- ∈ & ∉ The in symbol denotes membership in a set, and the not in symbol indicates that an element is not a member of the set.
- ⊆ & ⊂ The set inclusion sign (⊆) and proper subset sign (⊂) represent the relationship between sets, with the former allowing for equality and the latter not.
- ∪ & ∩ Union (∪) combines two sets into one with all unique elements, while intersection (∩) finds the common elements between two sets.
- \ The set difference is denoted by a backslash, resulting in a set of elements unique to the first set but not in the second.
- △ Symmetric difference, denoted by a triangle or circled minus, includes elements unique to each of the two sets.
- ¬ The negation symbol is used in logic to indicate the opposite of a statement.

- ∨ & ∧ The or operator returns true if at least one operand is true, while the and operator requires both operands to be true.
- ⊕ The exclusive or operator returns true if exactly one of the operands is true.
- ⊤ & ⊥ The tee and up tack represent logical constants for true and false values, respectively.
- ∀ & ∃ The universal quantifier asserts that a statement is true for all elements, while the existential quantifier asserts the existence of at least one element for which the statement is true.
- ⟺ The conditional operator denotes an implication between two statements, where the second is true if the first is true.
- ℕ, ℤ, ℚ, ℝ, ℂ, ℍ, 𝓸, ℵ The blackboard bold letters represent the sets of natural numbers, integers, rationals, reals, complexes, quaternions, octonions, and cardinalities of infinite sets, respectively.
- ' In Lagrange's notation, an apostrophe denotes the derivative of a function, with additional apostrophes for higher derivatives.
- ∫ The integral symbol represents an antiderivative or the definite integral, denoting the area under a curve or accumulation over an interval.
- ∧ The arrow is sometimes used to define a function without naming it, while function composition combines two functions.
- log The logarithm is the inverse of exponentiation, with subscripts denoting the base, and ln representing the natural logarithm with base e.
- lim The limit denotes the behavior of a function or expression as its input approaches a certain value.
- ℝ The fancy R denotes the real part of a complex number, while the fancy i denotes the imaginary part.
- Σ & Π The Greek letter sigma (Σ) is used for summation of a series, and capital Pi (Π) for the product of terms.
- ∞ The infinity symbol represents a value greater than any finite quantity, with Aleph (ℵ) and Fractur c representing different types of infinity and their cardinalities.
- ! The factorial operation multiplies a number by all positive integers smaller than itself.
- ⌊ & ⌈ The floor function returns the greatest integer less than or equal to a value, while the ceiling function returns the smallest integer greater than or equal to it.
- ‖ & ∞ The single and double lines represent divisibility and parallelism, with crossed lines indicating non-divisibility and non-parallelism.
- ⊥ The upside-down T represents perpendicularity and can also denote that two numbers are coprime.
- ∆ The bar over two points represents a line segment, an arrow over two points represents a ray, and a double-headed arrow represents an infinite line through both points.

Q & A

- **What does the plus sign universally represent?**

  -The plus sign universally represents the operation of addition.
- **What is the primary function of the minus sign?**

  -The minus sign primarily represents subtraction and can also denote negative numbers.
- **How can multiplication be represented besides the multiplication sign?**

-Multiplication can also be represented as a dot.

- **What does the division sign signify and how can it sometimes be written?**

-The division sign signifies division, the opposite of multiplication, and can sometimes be written as a slash.

- **What does the plus-minus sign denote and how can it be used?**

-The plus-minus sign denotes either plus or minus and can be used to denote a range of values.

- **What does the equals sign represent and what does it look like?**

-The equals sign, represented by two lines of equal length, is used to denote the equality between two expressions.

- **What does the not-equal sign indicate?**

-The not-equal sign indicates that two expressions are not equal.

- **What is the triple bar or equivalence sign commonly used to denote?**

-The triple bar or equivalence sign is commonly used to denote congruence in modular arithmetic.

- **What does the less than or equal sign indicate?**

-The less than or equal sign indicates that one value is smaller or equal to another.

- **What does the empty set symbol represent?**

-The empty set symbol represents a set that contains no elements.

- **How is the set inclusion sign used and what does it represent?**

-The set inclusion sign represents that one set is a subset of another set, and if a line is added, it emphasizes that the sets can be equal.

- **What does the union operation in set theory result in?**

-The union operation results in a set containing all unique elements from both sets being combined.

- **What is the purpose of the integral symbol in calculus?**

-The integral symbol denotes an antiderivative, which is the opposite of the derivative, and with subscript and superscript, it denotes a definite integral, representing the area under a curve or the accumulation of a quantity over an interval.

- **What does the absolute value of a number represent?**

-The absolute value of a number represents the distance of that number from zero on the number line.

- **What does the Greek letter sigma (∑) denote in mathematics?**

-The Greek letter sigma (∑) is used to denote the summation of a series of terms.

- **What is the difference between the natural logarithm and a common logarithm?**

-The natural logarithm, denoted as ln, represents a logarithm with base e, while a common logarithm, denoted as log without a subscript, represents the logarithm with base 10.

- **What does the limit denote in mathematics?**

-The limit is used to denote the behavior of a function or an expression as its input approaches a certain value.

- **What does the infinity symbol represent?**

-The infinity symbol represents a concept of unlimitedness, signifying a value that is greater than any finite quantity.

- **What is the difference between a proper subset and a subset in set theory?**

-A proper subset denotes that the sets are not equal, while a subset can be equal to the set it is being compared to.

How Infinity Works (And How It Breaks Math)

Summary

TLDRThis delves into the fascinating concept of infinity, exploring its mathematical significance through set theory and cardinality. It discusses Cantor’s groundbreaking work on different sizes of infinity and how arithmetic can be performed on infinite sets. The video also covers the use of limits in extending finite operations to infinity, as well as the historical and philosophical implications of infinity in mathematics. With engaging examples and thought-provoking insights, it provides a deep yet accessible introduction to one of the most mind-bending concepts in mathematics.

Takeaways

- 😃 Infinity isn't a simple number; it represents a concept of unbounded size or extent.
- 😃 Cardinality is the concept used to measure the size of sets, and infinite sets can have different cardinalities.
- 😃 The smallest infinite set is the set of natural numbers, with cardinality denoted as α_0 (aleph-null).
- 😃 Some sets, like the set of integers, may have the same cardinality as the natural numbers, making them countably infinite.
- 😃 The real numbers, on the other hand, are uncountable and have a cardinality larger than that of the natural numbers.
- 😃 Cantor's diagonalization proof shows that there are different sizes of infinity, and the real numbers are one such example.
- 😃 Operations on infinite cardinalities follow specific rules that differ from operations with finite numbers, but they are still consistent.
- 😃 The Continuum Hypothesis posits that there is no cardinality between α_0 and the cardinality of the real numbers, but it remains an unsolved problem.
- 😃 Infinity can be treated in calculus using the concept of limits, which help us reason about infinity without directly using infinite values in operations.
- 😃 The use of limits extends mathematical operations like addition, multiplication, and division to infinity in a manageable and consistent way.

- 😛 The understanding of infinity allows mathematicians to perform arithmetic on different sizes of infinity and explore their properties.

Q & A

- **What is the main challenge when dealing with infinity in mathematics?**

  -The main challenge is that infinity does not behave in intuitive ways, making it difficult to apply standard arithmetic operations like addition, subtraction, and multiplication directly to it. Mathematicians use specialized tools like set theory and limits to handle infinity in a consistent way.

- **How does the concept of cardinality relate to infinity?**

  -Cardinality refers to the size of a set, and when dealing with infinite sets, cardinality helps us compare the sizes of different infinite sets. For example, the cardinality of the natural numbers ($\aleph_0$) is different from the cardinality of the real numbers, which is larger ($\aleph_1$).

- **What is the Continuum Hypothesis and why is it important?**

  -The Continuum Hypothesis posits that there is no cardinal number between the cardinality of the natural numbers ($\aleph_0$) and the cardinality of the real numbers ($\aleph_1$). It's important because it highlights unresolved questions in set theory and has implications for the structure of the mathematical universe.

- **How can infinity be manipulated through cardinal arithmetic?**

  -Infinity can be manipulated through cardinal arithmetic by applying operations such as addition, multiplication, and exponentiation to cardinal numbers. For example, adding two infinities like $\aleph_0 + \aleph_0$ still results in $\aleph_0$, and raising 2 to the power of $\aleph_0$ leads to a larger infinity ($\aleph_1$).

- **Why is using limits crucial when dealing with infinity in calculus?**

  -Using limits is crucial because they allow us to approach infinity without directly dealing with it. Limits provide a way to evaluate infinite processes, such as sums or integrals, by examining the behavior of a function as its input approaches infinity, making it possible to work with infinity in a manageable way.

- **What does it mean that different infinite sets can have different sizes?**

  -It means that not all infinities are equal. Some infinite sets, like the set of natural numbers, have a smaller cardinality ($\aleph_0$) than other infinite sets, like the set of real numbers, which have a larger cardinality ($\aleph_1$). This distinction shows that there are different 'sizes' of infinity.

- **Can we apply standard arithmetic operations directly to infinity?**

-No, we cannot apply standard arithmetic operations directly to infinity because infinity doesn't follow the usual rules of arithmetic. Instead, mathematicians use tools like cardinal arithmetic and limits to work with infinity in a structured way.

- **How does pairing elements from different infinite sets help understand their cardinality?**

-Pairing elements from different infinite sets helps determine if they have the same cardinality. For example, pairing each natural number with an even number shows that both sets have the same cardinality, even though one seems to have 'half' the elements. This illustrates that infinity doesn't behave like finite numbers.

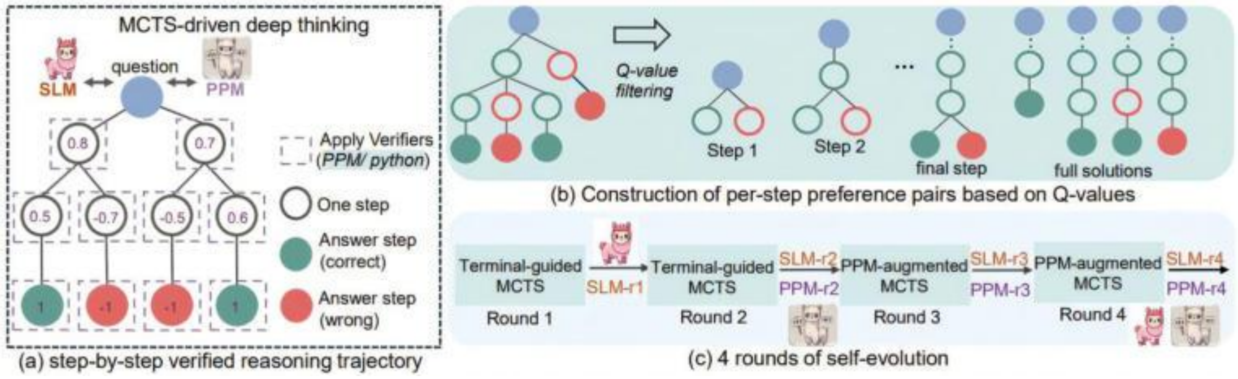
- **What role does set theory play in understanding infinity?**

-Set theory plays a central role in understanding infinity by providing a framework for comparing and classifying different types of infinity. It introduces the concept of cardinality, which allows mathematicians to compare the sizes of infinite sets and understand how different infinities relate to each other.

- **Why is the question of infinity still a source of philosophical debate?**

-The question of infinity is a source of philosophical debate because it challenges our intuitions about size and existence. Since infinity doesn’t conform to ordinary concepts of number and measurement, it raises questions about the nature of reality and how mathematical constructs like infinity relate to the real world.

Microsoft introduces rStar-Math, an SLM for math reasoning and problem solving



Overview of rStar-Math. Credit: arXiv (2025). DOI: 10.48550/arxiv.2501.04519

A team of math and AI researchers at Microsoft Asia has designed and developed a small language model (SLM) that can be used to solve math problems. The group has posted a paper on the *arXiv* preprint server outlining the technology and math behind the new tool and how well it has performed on standard benchmarks.

Over the past several years, multiple tech giants have been working hard to steadily improve their LLMs, resulting in AI products that have in a very short time become mainstream. Unfortunately, such tools require massive amounts of computer power, which means they consume a lot of electricity, making them expensive to maintain.

Because of that, some in the field have been turning to SLMs, which as their name implies, are smaller and thus far less resource intensive. Some are small enough to run on a local device. One of the main ways AI researchers make the best use of SLMs is by narrowing their focus—instead of trying to answer any question about anything, they are designed to answer questions about something much more specific—like math. In this new effort, Microsoft has focused its efforts on not just solving math problems, but also in teaching an SLM how to reason its way through a problem.

In developing its model, Microsoft made it in a way that allows for its use by other, larger models. An overall strategy that could be the wave of the future. New LLMs could soon be nothing more than an amalgam of many SLMs.

Notably, the announcement by Microsoft came not long after the debut of its Phi-4 SLM, which also serves to solve math problems.

rStar-Math does its work differently than Phi-4, the researchers note, by making use of Monte Carlo Tree Search—a reasoning method developed to mimic the way humans attack problems in a step-by-by process. They note that by using such an approach, their new SLM can break down a problem into its smaller parts as a way to figure out how to solve a particular problem. They also note that rStar-Math shows its work by outputting its thought process in both Python code and natural language.

The team also noted that rStar-Math has already scored well on several benchmarks. And according to a post on Hugging Face, the team plans to make the code and data publicly available on GitHub.

More information: Xinyu Guan et al, rStar-Math: Small LLMs Can Master Math Reasoning with Self-Evolved Deep Thinking, *arXiv* (2025). DOI: 10.48550/arxiv.2501.04519

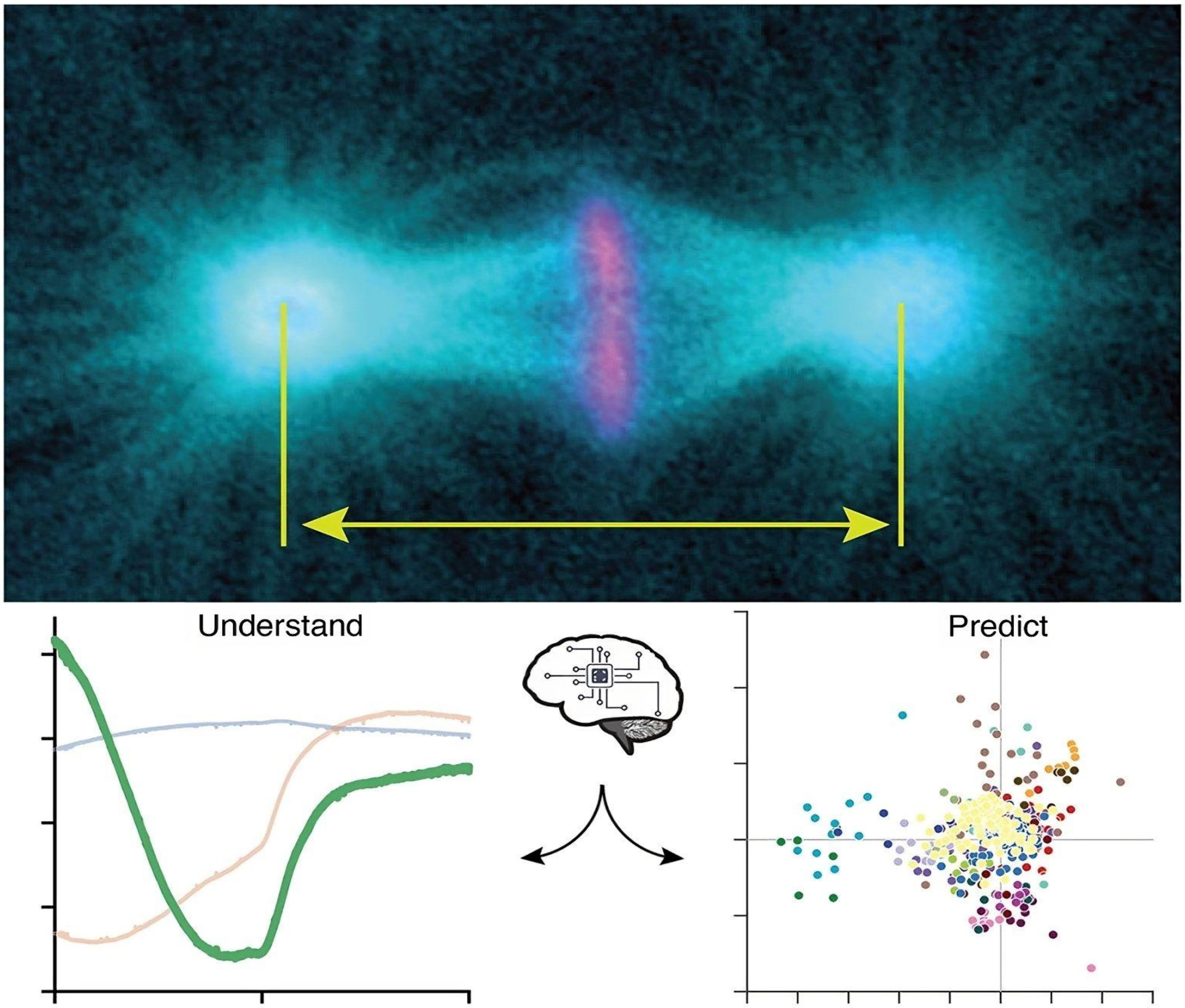
huggingface.co/papers/2501.04519

Mathematical analysis to better understand the mechanisms of cell division

Published October 16, 2024

In an article published in the journal *PLoS Computational Biology*, scientists used a data science approach to study the variations observed between different cells in spindle formation.

This structure allows the migration of chromosomes during cell division, ensuring its fidelity. They showed in particular that three parameters are sufficient to explain this variability.



A data science approach to study cell variability

Each cell is different from its neighbor. Quantifying this variability is essential because it contains important information about underlying cellular mechanisms.

Above, microtubules marked in cyan and chromosomes in magenta, along with the mitotic spindle poles, including centrosomes, structures tracked in our analyses. The image is taken in late metaphase.

Below left, the three archetypes, with the less expected third archetype also visible. These archetypes represent typical spindle elongations over time during metaphase and anaphase.

Below right, a classification showing the dosage of each archetype to reproduce the elongations under some genetically perturbed conditions. Each color corresponds to a given gene. When points are close together, this suggests similar phenotypes, indicating that these genes may belong to the same signaling pathway or mechanism.
© Jacques Pécréaux and Laurent Chesneau

In an article published in the journal *PloS Computational Biology*, the scientists focused on the length of the mitotic spindle, which forms to allow the migration of chromosomes during cell division, to study this variability. This measurement is commonly used to indicate whether the division is proceeding correctly.

They used the nematode worm, *Caenorhabditis elegans*, to conduct the study. In this model, cell divisions are well characterized and reproducible. Additionally, its genome can be easily manipulated to precisely control the relationship between phenotype and genotype.

The scientists compiled elongation curves from 1,500 cells under control and genetically perturbed conditions to represent the variety of possibilities. To carry out an unbiased analysis, they based their approach solely on the data. The variability descriptors were automatically extracted.

With this method, they obtained two descriptors similar to those already known: spindle length and elongation rate in anaphase (the phase of mitosis where chromosomes reach the spindle poles).

However, they also discovered a new descriptor: shortening at the end of metaphase (the phase of mitosis where chromosomes are gathered in the center of the spindle) — present in all conditions. Such a phenotype had previously been limited to cells with defective chromosome attachments.

An analysis that highlights fundamental mechanisms.

These three descriptors account for 95% of the variability, suggesting that the complex choreography of the spindle relies on just a few basic mechanisms. This also limits the possible phenotypes, pointing to mechanisms ensuring the robustness of the division.

Furthermore, the scientists demonstrated that the final spindle length in anaphase, important for determining the fate of daughter cells, is already set at the end of metaphase, despite the spindle being completely rearranged between the two phases. This reveals an unexpected interdependence between metaphase and anaphase spindles.

Moreover, the same descriptors explain the variability under genetically perturbed conditions. This suggests that no new mechanisms appear in defective cells; only the contributions of existing mechanisms change.

Ultimately, these findings shed light on the fundamental mechanistic principles governing mitotic spindles and their robustness. This will help to identify the mechanisms by which cancer cells manage to divide despite accumulated defects and antimitotic treatments.

Beyond these initial results on the mitotic spindle, this work also provides a practical tool for quantifying phenotypes in other cellular contexts. Through the use of artificial intelligence, this method will suggest new candidate genes involved in cell division mechanisms. This is a significant step toward enhancing our understanding and identifying mechanism actors, to potentially develop them into future therapeutic targets.

References:

Le Cunff Y, Chesneau L, Pastezeur S, Pinson X, Soler N, Fairbrass D, et al. (2024). Unveiling inter-embryo variability in spindle length over time: Towards quantitative phenotype analysis.
PLoS Comput Biol 20(9): e1012330. https://doi.org/10.1371/journal.pcbi.1012330

DECEMBER 23, 2024

Algebraic geometry offers fresh solution to data center energy inefficiency

by Kelly Izlar, Virginia Tech

Data storage centers are driving power use and threatening climate goals. Virginia Tech mathematicians using geometric structures to develop a smarter way to store and recover data. Credit: Kathy Acosta for Virginia Tech

The manic pace of sharing, storing, securing, and serving data has a manic price—power consumption. To counter this, Virginia Tech mathematicians are leveraging algebraic geometry to target the inefficiencies of data centers.

"We as individuals generate tons of data all the time, not to mention what large companies are producing," said Gretchen Matthews, mathematics professor and director of the Southwest Virginia node of the Commonwealth Cyber Initiative. "Backing up that data can mean replicating and storing twice or three times as much information if we don't consider smart alternatives."

Instead of energy-intensive data replication, Matthews and Hiram Lopez, assistant professor of mathematics, explored using certain algebraic structures to break the information into pieces and spread it out among servers in close proximity to each other. When one server goes down, the algorithm can poll the neighboring servers until it recovers the missing data.

Mathematicians have known since the 1960s that polynomials can be used to store information. But in the last decade, researchers have discovered how to build special polynomials that can store data in convenient configurations for applications like recovering missing information locally.

"Turns out there's some beautiful mathematical structures that were developed over the years that can provide a better way to store data and serve additional requests," Matthews said.

Their presentation of a new method for storing and serving data was featured in an invited review article in *IEEE BITS*.

Matthews and Lopez's work comes at a time when demand for electricity is surging across the nation: Grid planners forecast peak demand to increase by 38 gigawatts through 2028. The anticipated increase is largely due to the construction of new data centers, many of which are under construction or being planned in Virginia.

Besides targeting inefficiencies in how server farms store data, the method also addresses energy use associated with how data center algorithms search for requested information.

"All of these structures are tied to the physical world, and they are subject to space and time," said Lopez. "It takes energy to find and retrieve information."

If too many people try to access the same information at the same time, the system will fail and result in what's colloquially referred to as "breaking the internet." When a selfie or a video goes viral, every request to see or share the content pings some of the servers that store the backups. At some point, there are no copies available to be viewed—and the server crashes.

Matthews and Lopez's technique, which uses an error-correcting code, improves data access and storage in two main ways:

- Servers don't have to store complete duplicates of any information, which means they now have more storage space.
- During server failure or data erasure, the algorithm doesn't have to expend energy searching the entire network to recover the missing information—it simply needs to see what the neighboring servers have in store.

In a subsequent research project and publication for *Designs, Codes and Cryptography*, Matthews and collaborators noted that the underlying structure of a particularly famous class of error-correcting codes, called Reed Muller codes, allows for the recovery of missing information naturally.

This type of application demonstrates how deep mathematics can be relevant and impactful to issues that our society is facing, not just here in the commonwealth, but as a nation and as a global community, Matthews said.

"Improving the systems and processes we already have in place can help us meet our goals for sustainable growth," Matthews said.

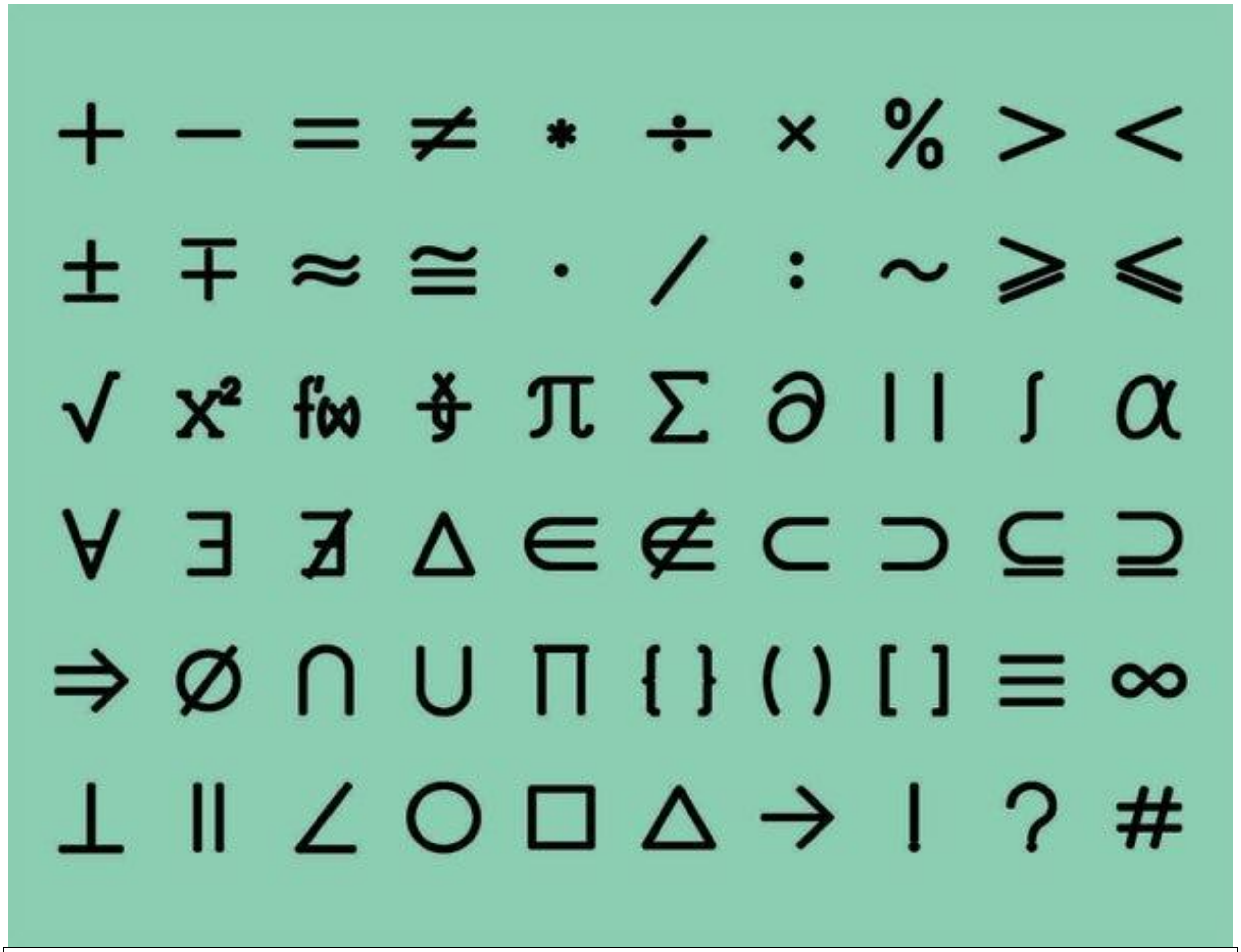
More information: Kathryn Haymaker et al, Mathematical LoRE: Local Recovery of Erasures Using Polynomials, Curves, Surfaces, and Liftings, *IEEE BITS the Information Theory Magazine* (2024). DOI: 10.1109/MBITS.2024.3359988

Kathryn Haymaker et al, Algebraic hierarchical locally recoverable codes with nested affine subspace recovery, *Designs, Codes and Cryptography* (2024). DOI: 10.1007/s10623-024-01510-x
Provided by Virginia Tech

FEBRUARY 7, 2025

The Wild and Contentious History of Mathematical Symbols

A mathematician has uncovered the stories behind the symbols used in math



Mathematics

War in Europe is a staple topic in the study of history, but there’s one major conflict most history books won’t teach you—the battle of the equals sign, “=.” These two parallel lines were, in fact, the source of major conflict between European mathematicians in the mid 1500s. This is just one of many untold histories in *The Language of Mathematics: The Stories behind the Symbols* by author and mathematician Raúl Rojas. In it, Rojas explores the complex, and sometimes uncertain, history of mathematical symbolism.

Another debate has been raging for centuries, for instance, over who invented the symbol for zero, “0.” Over the years, competing camps have debated adopting one or another notation for many different aspects of mathematics. Rojas’s tale guides us along the historical arc of mathematics, intertwining its evolution with the cultural, philosophical and practical needs of the societies that shaped and relied on it.

Scientific American spoke to Rojas about this history, the deeply engaging humanity of mathematics and the egos at play in defining the mathematical language we take for granted today.

[*An edited transcript of the interview follows.*]

What inspired you to write this book about the stories behind these symbols?

I started teaching in 1977, and across my nearly 50 years, I noticed that students were always interested in the history of mathematics. When you teach linear algebra or calculus, it’s important to tell students about the people who developed the concepts and how those concepts came to be. I started doing seminars on the history of mathematical notation and had every student study one symbol and explain its origin. I found that those students who are falling asleep in class suddenly wake up when you add a human story behind the abstract symbols.

Throughout the book, you discuss symbols that ultimately failed to become the standard en route to the notation we know today. How were these things decided?

One of the interesting things about the history of mathematical notation is its regional variation over the centuries. There was one kind of notation in Italy, another in Germany, the U.K. and France. All these different regions were producing symbols, and with the advent of the printing press, there was an explosion of proposals. So how did it happen that a single symbol could become standardized?

One good example is the symbol of equality, “=.” This relation was mostly expressed with words in the beginning. Later [René Descartes in France started using the rotated Taurus symbol, “∝,” while [Gottfried Wilhelm] Leibniz in Germany used a wedgelike shape. And [before Descartes and Leibniz] Robert Recorde in [the U.K.] invented the equality sign we use today, though in an elongated form. Mathematicians found themselves in a kind of battle over arithmetical symbols based on popularity. A notable contest was between “+” and “–” versus “p” and “m,” which the Italians preferred for denoting operations. Eventually, the plus and minus signs became universal, as did the English symbol for equality, but only after decades of famous mathematicians competing in these popularity contests to set the trends.

Is there a particular symbol in the history of mathematics that significantly influenced how we think about abstract concepts?

There is one symbol, which has an incredibly long history that has not yet been fully written: “0.” How did it arise? We know it was used by the Babylonians, but they didn’t write a “0” as we know it. They worked with a positional base-60 system and simply left a blank where we would write 0 today. This was their natural way of showing zero: if it’s nothing, then you don’t have to write anything.

Later, through the conquests of Alexander the Great, the Greeks took the positional number system to India, where we believe the Hindu culture developed the first representation of “0.” There’s a friendly competition between anthropologists working to find the oldest instances of “0” in writing. Every five or six years, someone finds an older engraving. It’s fascinating because this simple symbol we use every single day without thinking about it has a history that encompasses … thousands of years.

You describe Gerhard Gentzen’s “for all” symbol (∀) as a “cubist tear flowing from an eye that Picasso could have painted.” What’s the story behind that notation?

Gentzen’s life is deeply tragic to me. He was an exceptional mathematician who, like many others in Nazi Germany, compromised with the regime. Although he was never a political person, he became a member of the Nazi Party and later joined the SS—the most criminal arm of the regime. Absorbed in his work, he made these compromises to advance his career. Even after the war, he expressed no guilt, claiming that he was neither a soldier nor doing anything wrong. He had taken a position at the University of Prague, however, displacing others under Nazi occupation. Ultimately, he chose not to flee after the war, was captured, and died of starvation in prison.

There is no excusing his actions, but his life remains tragic from beginning to end, especially when considering what he might have accomplished had he taken a different path. It’s fascinating that such a simple symbol—this upside-down “A”—carries such a complex and poignant history.

What do you hope readers—especially those outside the math community—might take away from your book?

It’s important to understand that mathematics is a historical process, just like any social science or politics. Mathematics didn’t arise complete and finished through the work of just one mathematician; it has a cultural history that spans many years. For centuries, we’ve been looking at the sky or computing. In school, they teach addition and multiplication but rarely explain the origins or history of the symbols. This vast history is untold, but the excitement of doing mathematics comes from this knowledge that you are building on a framework developed by fascinating people over thousands of years.

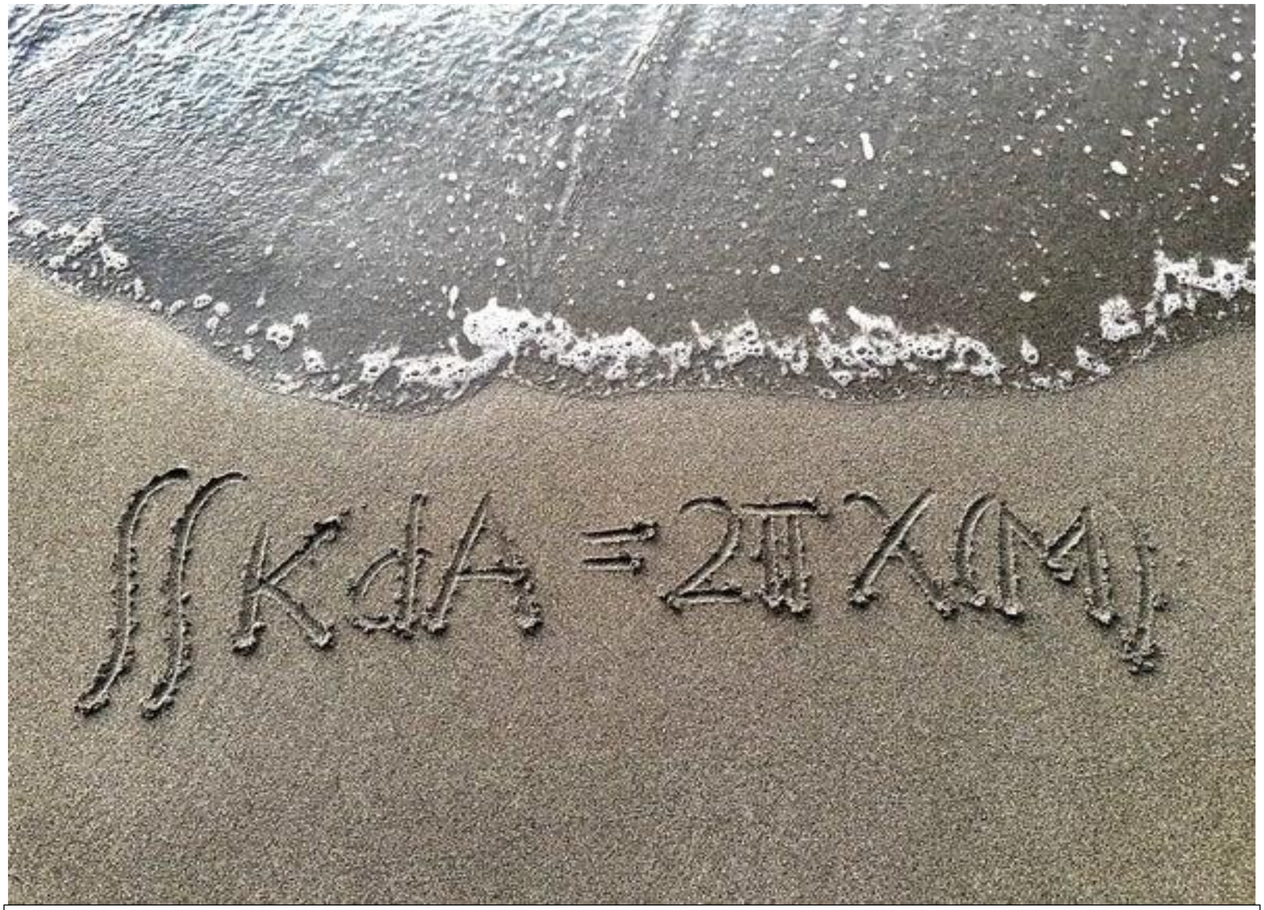
JUNE 20, 2024

8 MIN READ

These Are the Most Beautiful Equations, according to Mathematicians

Mathematicians picked the most dazzling, thought-provoking and compelling equations they know

BY RACHEL CROWELL EDITED BY CLARA MOSKOWITZ



Ailana Fraser

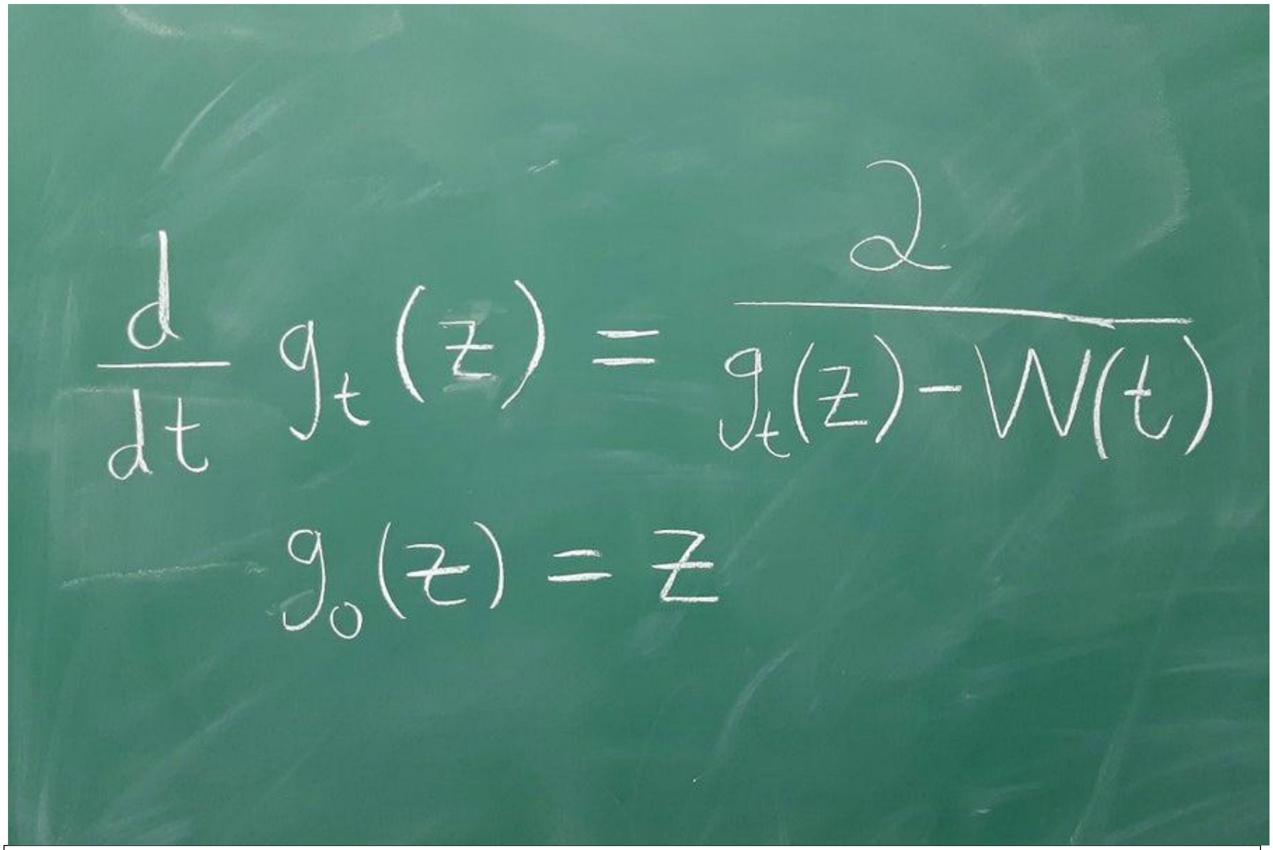
Mathematics

To mathematicians, equations are art. Just as many are moved by a painting or piece of music, to those who appreciate and understand math, expressions of numbers, variables, operations and relations between quantities can be just as compelling.

As is the case with artistic beauty, mathematical beauty is in the eye of the beholder. One mathematician may prize simple-to-state, succinct equations, while another may favor the opposite. And just as some favor modern art while others prefer medieval works, both ancient and contemporary equations are admired for their cleverness, power and possibility.

Here are some mathematicians’ selections of the most beautiful math equations, as well as their own words on what factors make them so exquisite.

LOEWNER DIFFERENTIAL EQUATION



$$\frac{d}{dt} g_t(z) = \frac{2}{g_t(z) - W(t)}$$

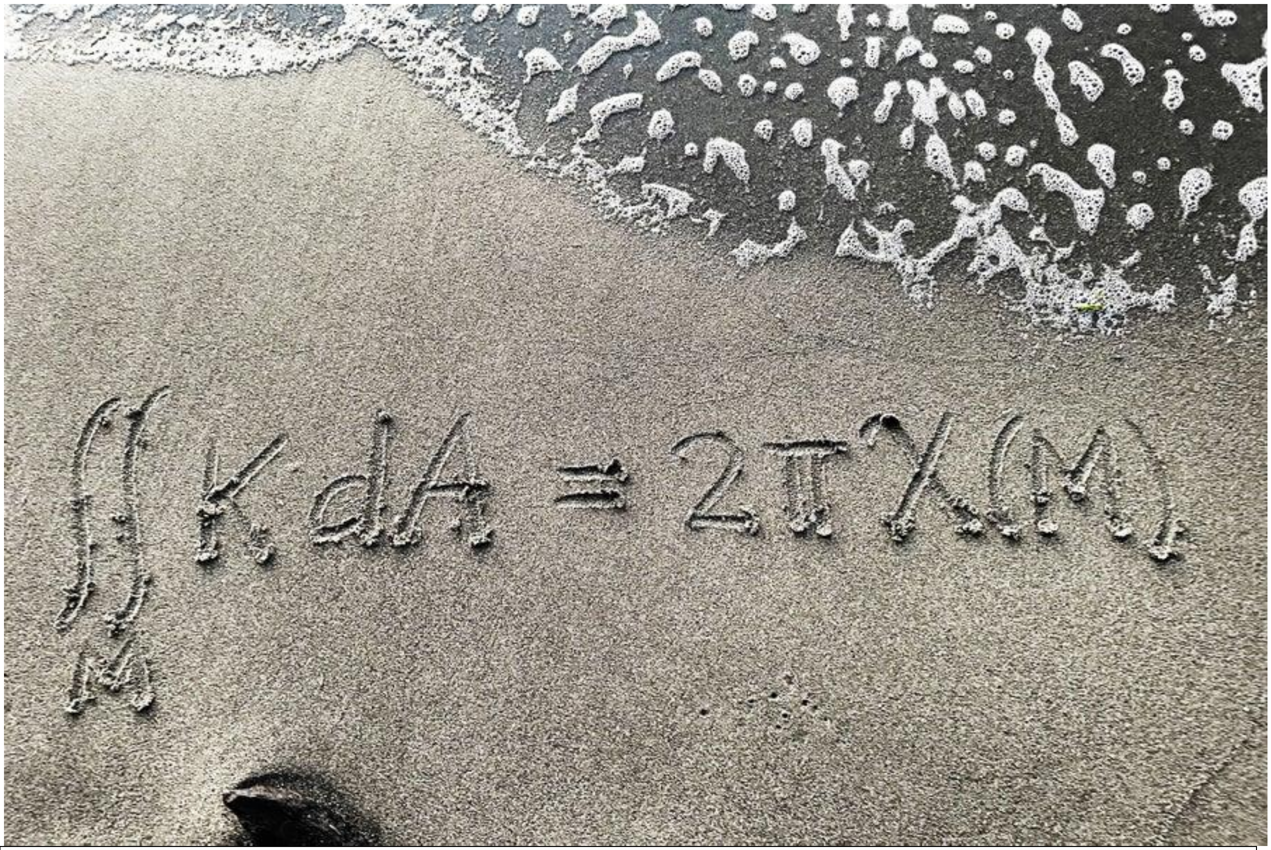
$$g_0(z) = z$$

Ewain Gwynee

Some equations are beautiful because they reveal unexpected relationships between different subjects. The Loewner differential equation, introduced by Charles Loewner in 1923, describes the time evolution of a family of conformal (angle-preserving) functions defined on subsets of the complex plane. The input for the equation is a continuous function $W(t)$, called the driving function.

Nearly 80 years later, in 1999, Oded Schramm discovered that the solution to the Loewner equation has special symmetries when the driving function is taken to be Brownian motion, a random function that is a central object of study in probability theory. Building on Schramm’s discovery, it was proven that the solution to the Loewner equation for this choice of driving function, called Schramm-Loewner evolution, describes the large-scale behavior of various critical models in two-dimensional statistical mechanics. This application of the Loewner equation in probability theory was completely unexpected prior to Schramm’s work. It revolutionized the mathematical study of statistical mechanics and has led to some of the most exciting breakthroughs in mathematics in the past two decades. —*Ewain Gwynne, University of Chicago*

GAUSS-BONNET EQUATION

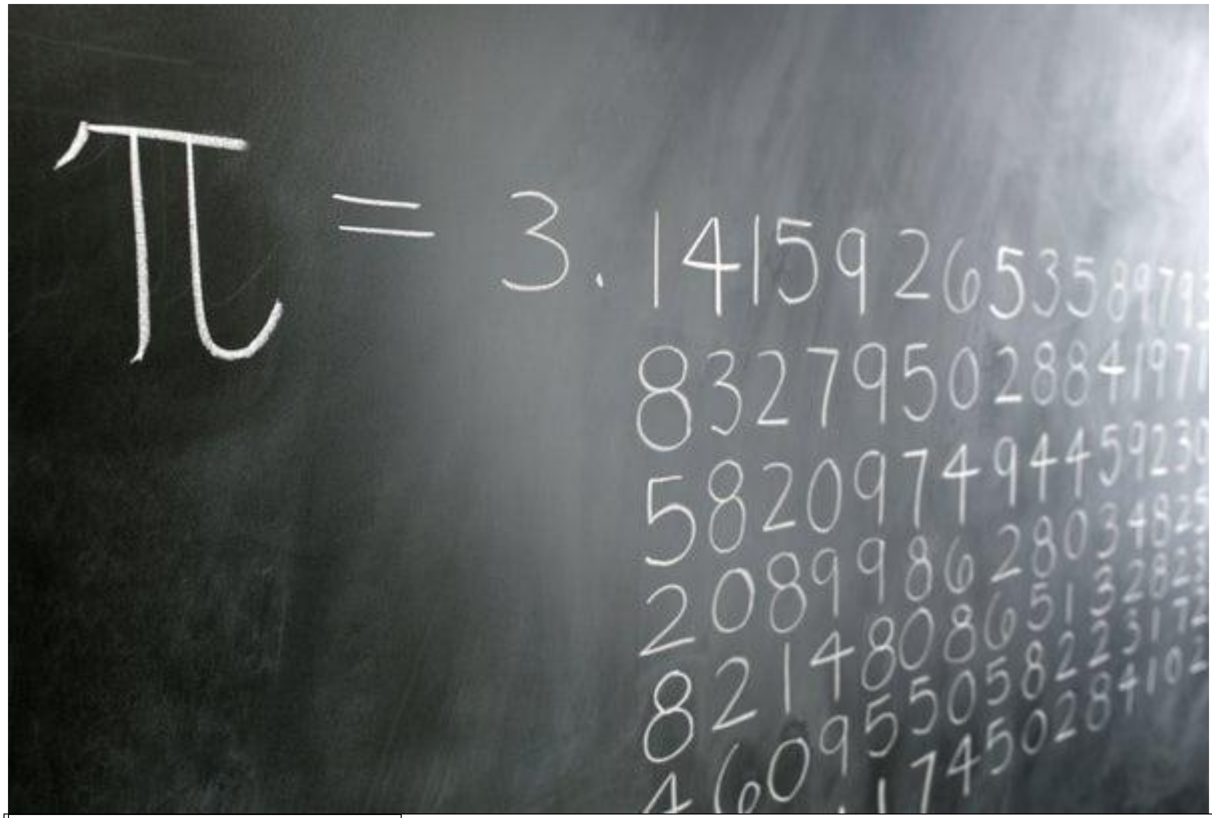


Ailana Fraser

The Gauss-Bonnet formula is a beautiful equation in differential geometry that asserts the equality of the integral of the Gauss curvature over a surface and a constant multiplied by the Euler characteristic of the surface. The equation is remarkable because it relates two very different quantities: the curvature of the surface on the one hand and the topology of the surface on the other hand. The curvature of a surface at a point is a measure of the bending of the surface at that point, or how much the surface deviates from being a plane. The Euler characteristic is a global topological invariant of the surface that describes the topological structure of the surface regardless of how it is bent. The equation is surprising because, for example, it implies that if you continuously deform a surface, the total curvature will remain unchanged. Versions of the Gauss-Bonnet equation were first formulated by Carl Friedrich Gauss and Pierre Ossian Bonnet in the first half of the 19th century, and it remains one of the most beautiful and striking equations in geometry. Some beautiful aspects of this equation are the simplicity of its expression and the profoundness of what it says. *—Ailana Fraser, University of British Columbia*

What Is Pi, and How Did It Originate?

Steven Bogart, a mathematics instructor at Georgia Perimeter College, answers



Jeffrey Coolidge *Getty Images*

The Sciences

Succinctly, pi—which is written as the Greek letter for p, or π—is the ratio of the circumference of any circle to the diameter of that circle. Regardless of the circle's size, this ratio will always equal pi. In decimal form, the value of pi is approximately 3.14. But pi is an irrational number, meaning that its decimal form neither ends (like 1/4 = 0.25) nor becomes repetitive (like 1/6 = 0.166666...). (To only 18 decimal places, pi is 3.141592653589793238.) Hence, it is useful to have shorthand for this ratio of circumference to diameter. According to Petr Beckmann's *A History of Pi*, the Greek letter π was first used for this purpose by William Jones in 1706, probably as an abbreviation of periphery, and became standard mathematical notation roughly 30 years later.

Try a brief experiment: Using a compass, draw a circle. Take one piece of string and place it on top of the circle, exactly once around. Now straighten out the string; its length is called the circumference of the circle. Measure the circumference with a ruler. Next, measure the diameter of the circle, which is the length from any point on the circle straight through its center to another point on the opposite side. (The diameter is twice the radius, the length from any point on the circle to its center.) If you divide the circumference of the circle by the diameter, you will get approximately 3.14—no matter what size circle you drew! A larger circle will have a larger circumference and a larger radius, but the ratio will always be the same. If you could measure and divide perfectly, you would get 3.141592653589793238..., or pi.

Otherwise said, if you cut several pieces of string equal in length to the diameter, you will need a little more than three of them to cover the circumference of the circle.

Pi is most commonly used in certain computations regarding circles. Pi not only relates circumference and diameter. Amazingly, it also connects the diameter or radius of a circle with the area of that circle by the formula: the area is equal to pi times the radius squared. Additionally, pi shows up often unexpectedly in many mathematical situations. For example, the sum of the infinite series

1 + 1/4 + 1/9 + 1/16 + 1/25 + ... + 1/n2 + ... is $\pi^2/6$

The importance of pi has been recognized for at least 4,000 years. *A History of Pi* notes that by 2000 B.C., "the Babylonians and the Egyptians (at least) were aware of the existence and significance of the constant π," recognizing that every circle has the same ratio of circumference to diameter. Both the Babylonians and Egyptians had rough numerical approximations to the value of pi, and later mathematicians in ancient Greece, particularly Archimedes, improved on those approximations. By the start of the 20th century, about 500 digits of pi were known. With computation advances, thanks to computers, we now know more than the first six billion digits of pi.

Mathematician solves two long-standing problems scientists couldn't crack

One of Pham's major achievements was solving the Height Zero Conjecture, a problem first proposed in 1955 by the renowned mathematician Richard Brauer. (CREDIT: Pham Tiep)© The Brighter Side of News

Two breakthroughs in mathematics could reshape how researchers understand symmetries in nature and the behavior of complex systems. These advances, led by Pham Tiep, a professor at Rutgers University, have implications across physics, chemistry, computer science, and economics.

Tiep’s work centers on representation theory, a branch of algebra that translates abstract symmetries into numerical form using matrices. This mathematical framework underpins a wide range of applications, from encryption algorithms to quantum mechanics. By pushing the boundaries of this field, Tiep has reinforced Rutgers' position as a global leader in advanced mathematical research.

One of his major achievements was solving the Height Zero Conjecture, a problem first proposed in 1955 by the renowned mathematician Richard Brauer. For decades, this conjecture stood as a daunting challenge in the study of finite groups. Mathematicians had struggled to prove it, but Tiep and his collaborators found a way forward, breaking a barrier that had long frustrated researchers.

Conjectures, by their nature, demand proof before they can be accepted as fact. Tiep described the challenge, saying, “A conjecture is an idea that you believe has some validity. But conjectures have to be proven. I was hoping to advance the field. I never expected to be able to solve this one.” His success was a triumph of perseverance and deep mathematical insight.

Brauer’s work from the mid-20th century outlined a series of unresolved problems that have guided mathematicians for generations. Tiep and his colleagues have followed this blueprint, tackling some of the field’s toughest puzzles. Reflecting on Brauer’s brilliance, he remarked, “Some mathematicians have this rare intellect. It’s as though they came from another planet or from another world. They are capable of seeing hidden phenomena that others can’t.”

For over a decade, Tiep pursued this problem, driven by curiosity and a desire to advance knowledge. His solution not only resolves a long-standing mystery but also lays the groundwork for further discoveries. The techniques developed in his proof may offer fresh insights into other major mathematical challenges.

Among the problems that could benefit from these advances are those posed by legendary mathematicians John Thompson and Alexander Lubotzky. The tools forged in solving the Height Zero Conjecture may now help mathematicians crack open new doors in algebra and beyond.

The second major advancement relates to the Deligne-Lusztig theory, another key aspect of representation theory. Tiep’s solution addresses issues involving the trace of a matrix, which is the sum of its diagonal elements. This seemingly technical detail is crucial for understanding the deeper structures within representation theory.

Tiep's research in this area was published in two papers—one in Inventiones mathematicae and the other in Annals of Mathematics. These contributions provide new tools for analyzing traces, which could lead to further progress in various branches of mathematics.

“Tiep’s high-quality work and expertise on finite groups has allowed Rutgers to maintain its status as a top worldwide center in the subject,” said Stephen Miller, Chair of the Department of Mathematics at Rutgers. Miller pointed out that one of the major achievements in 20th-century mathematics was the classification of finite simple groups, a field in which Rutgers played a pivotal role. “Through his amazing stretch of strong work, Tiep brings international visibility to our department,” Miller added.

Tiep's work, while grounded in abstract mathematical concepts, has far-reaching applications. Representation theory is vital in studying the symmetry of molecules, creating encryption algorithms, and developing error-correcting codes for communication systems.

The methods he employs in his research highlight the power of pure thought and collaboration. Tiep’s process involves pen and paper—he often jots down formulas and logical chains of reasoning. Despite the simplicity of his tools, his results have revolutionized understanding in his field.

Collaboration is central to Tiep’s success. His work on the Height Zero Conjecture was a joint effort with several international colleagues, including Gunter Malle from Germany, Gabriel Navarro from Spain, and Amanda Schaeffer Fry, a former student now at the University of Denver.

For the Deligne-Lusztig theory breakthrough, he worked with Robert Guralnick of the University of Southern California and Michael Larsen of Indiana University. Together, they tackled the problem of traces, producing solutions that promise to yield further advancements.

“Tiep and coauthors have obtained bounds on traces which are about as good as we could ever expect to obtain,” Miller said, referring to the rigor of their findings. He emphasized that progress in such a mature subject is rare, but the potential applications are broad and impactful.

Pham Tiep, a professor at Rutgers University. (CREDIT: CC BY-SA 4.0)© The Brighter Side of News

Unlike many researchers in the physical sciences who rely on complex equipment, much of Tiep's progress comes through careful thought and collaboration with his peers, sometimes in person and sometimes via Zoom. He explains that his best ideas often emerge when he least expects them.

"Maybe I'm walking with our children or doing some gardening with my wife or just doing something in the kitchen," he shared. His wife, he says, can always tell when he's thinking about math.

By breaking through these long-standing problems, Tiep has not only advanced the field of mathematics but also highlighted the value of persistence and collaboration. His work opens the door to future discoveries that may further transform our understanding of the natural and mathematical world.

GEOMETRY

'Once in a Century' Proof Settles Math's Kakeya Conjecture

The deceptively simple Kakeya conjecture has bedeviled mathematicians for 50 years. A new proof of the conjecture in three dimensions illuminates a whole crop of related problems.

If you rotate a needle around in all directions, what is the minimum volume that can be carved out?

Introduction

March 14, 2025

Consider a pencil lying on your desk. Try to spin it around so that it points once in every direction, but make sure it sweeps over as little of the desk's surface as possible. You might twirl the pencil about its middle, tracing out a circle. But if you slide it in clever ways, you can do much better.

"It's just a problem about how straight lines can intersect one another," said Jonathan Hickman(opens a new tab), a mathematician at the University of Edinburgh. "But there's such an incredible richness encoded in it — an incredible array of connections to other problems."

For five decades, mathematicians have sought the best possible solution to the three-dimensional version of this challenge: Hold a pencil in midair, then point it in every direction while minimizing the volume of space it moves through. This straightforward problem has eluded some of the greatest living mathematicians, and it lurks beneath a host of open problems.

Now, the hunt for a solution appears to be over. In a paper recently posted on the scientific preprint site arxiv.org, Hong Wang(opens a new tab) of New York University's Courant Institute and Joshua Zahl(opens a new tab) of the University of British Columbia have proved the three-dimensional Kakeya conjecture(opens a new tab) — they've established an absolute limit to how small such a pattern of movements can be.
"This thing doesn't need hyping up," said Nets Katz(opens a new tab), a mathematician at Rice University. "It's a once-in-a-century kind of result."

A Thickening Plot

In 1917, Sōichi Kakeya posed the problem, but with an infinitely thin pencil. He found a way of sliding the pencil that covered less area than the instinctual circular motion.

Kakeya wondered how small an area the pencil could possibly sweep. Two years later, the Russian mathematician Abram Besicovitch found the answer: a complicated set of narrow turns that, counterintuitively, covers no space at all.

That more or less settled the question until 1971, when Fefferman was studying something apparently unrelated to twirling lines: the Fourier transform, a foundational mathematical tool that lets you reimagine any mathematical function as a combination of waves. In Fefferman's work, a tweaked version of Kakeya's problem kept coming up. In this case, the pencil has a thickness and twirls in three dimensions. Here, Kakeya's question becomes the following: As you change the width of the pencil, how does it affect the volume of space that it traces out?

Mathematicians prefer to picture this problem in a slightly different (but equivalent) way. Instead of moving a pencil around in space, imagine every position in the pencil's trajectory, all at once. What you get is a configuration of ghostly, overlapping tubes pointing everywhere, called a Kakeya set. You can slide the tubes around, but you can't rotate them. Your goal is to form a configuration with the most overlap.

Hong Wang, a mathematician at the Courant Institute at New York University, said the proof will open up new vistas in mathematics. “It needed to be done,” she said.
Rickinasia/Wikimedia Commons

Even the Kakeya set that overlaps the most has to take up some space, Fefferman found. That minimum volume depends on how thick the tubes are. Mathematicians quantify the relationship between the tubes’ thickness and the volume of the set using a number called the Minkowski dimension. The smaller the Minkowski dimension, the more you can reduce the set’s volume by thinning the tubes slightly.

The three-dimensional Kakeya conjecture says that a set’s Minkowski dimension must be three. This constitutes a very weak relationship — if you halve the tubes’ thickness, for instance, you will only remove a sliver of the volume at most.

Yet even that mild constraint turned out to be nearly impossible to prove.

Baby Steps

In 2022, five decades after the modern Kakeya conjecture was formulated, Wang and Zahl took a significant step forward. Following a program that Katz and Terence Tao(opens a new tab) had laid out back in 2014, they examined a pesky class of Kakeya sets. Their proof showed that every set in that particular class had a dimension of three. (The proof applies to both the Minkowski dimension and a closely related concept called the Hausdorff dimension.) With that annoying group set aside, they now had to show that the dimension was three for all the other Kakeya sets.

Sōichi Kakeya posed the problem that would bear his name in 1917, when he was 31.
Courtesy of University of Tokyo

Their approach was to go step-by-step. They would first examine a narrow range of Minkowski dimensions — say, 2.5 to 2.6 — and try to show that no Kakeya set could be in that range. If they could prove this for every interval up to three, they’d prove the Kakeya conjecture.

Fortunately, Wang and Zahl didn’t have to start from zero. Tom Wolff proved in 1995 that no three-dimensional Kakeya set has a Hausdorff or Minkowski dimension below 2.5. But they needed a way to prove that a dimension between 2.5 and, say, 2.500001, was also impossible. Then they could repeat that argument to get a bound of 2.500002, and so on. Each time, they would essentially be showing that no Kakeya sets exist within that tiny increment.

In practice, they didn't actually have to tediously prove each of these millions of increments one by one. They just needed to prove the first increment, so long as they could show that one bound implies the next, slightly larger one. Then they had to show that their argument worked no matter where they began. That would be enough to show that the bound can be walked up all the way to three.

But unlike in 2022, when they used Katz and Tao's strategy, they had no road map to follow. They turned to a special property called graininess.

In 2014, Larry Guth(opens a new tab), a mathematician at the Massachusetts Institute of Technology, had proved that any counterexample to the Kakeya conjecture needed to be "grainy." In a grainy set, there are many small 3D sections where lots of tubes overlap. Each of these "grains" is about one tube thick and a few times wider, but not nearly as long, with many tubes passing through it lengthwise.

Wang and Zahl realized they could eschew the tubes entirely and deal with these simpler grains. They found that it was easier to enumerate and calculate the various ways the grains could overlap.

Joshua Zahl, a mathematician at the University of British Columbia, co-authored the new proof.
Paul Joseph

And even in cases where the grains all conspired to provide maximum overlap, they found, the number of grains intersecting any given point couldn't be too big. Starting from the 2.5 bound, they were able to prove that the grains couldn't overlap enough to result in a dimension slightly above that bound either. Then, starting from the higher bound, they showed that the same computational steps could be applied to nudge the bound even higher. And so on.

"It's like perfecting a perpetual-motion machine. It's magical," Tao said. "They're getting more at the output than the input." Their machine took them all the way to a Minkowski (and Hausdorff) dimension of three, proving the three-dimensional Kakeya conjecture.

Tower of Dreams

The conjecture's resolution is a seismic shift for the field of harmonic analysis, which studies the details of the Fourier transform.

A tower of three monumental conjectures in harmonic analysis rests atop the Kakeya conjecture. Each story in the tower needs to be sturdy for the stories above it to stand a chance themselves. If the Kakeya conjecture had been proved false — if Wang and Zahl had found a counterexample — the entire tower would have come tumbling down.

But now that they've proved it, mathematicians might be able to work their way up the tower, using Kakeya to build up proofs of these successively more ambitious conjectures. "All these problems that [mathematicians] dreamed about someday solving, they all look approachable now," Guth said.

That process has already begun. Wang recently co-authored a separate paper reducing the next conjecture in the tower to a stronger version of the Kakeya conjecture, a step toward bridging the two levels.

It's also a dimensional leap for this entire area of math that's been somewhat stuck in 2D. "People understood what's going on [in Kakeya-adjacent problems] really well in two dimensions, but we lacked the tools to study higher dimensions," Wang said. "So I feel like this was necessary. It needed to be done."

The four-dimensional Kakeya conjecture remains open, with a tower of four-dimensional conjectures above it as well. New difficulties will arise, Guth said, but he thinks that the jump from two dimensions to three was the hardest, and that Wang and Zahl's proof can likely be adapted to that tower, and beyond.

"When I got excited about the Kakeya problem as a younger mathematician, it just felt so simple and geometrical that it was surprising to me that it was hard," Guth said. Years later, Wang, his doctoral student, was motivated by the same deceptive simplicity.

“You have these concrete things you can visualize. It’s not as scary as other math theories,” Wang said. “I just wanted to understand why it’s hard.”

Now, thanks to Wang and Zahl’s efforts, that understanding is closer than ever. “I really think there’s a critical mass of ideas to really revolutionize the whole field coming from here,” Hickman said. “It’s a very, very exciting time.”