Unified Theory of Low and High-Temperature Superconductivity

Liu, Jerry Z. Ph.D.
ZJL@CS.Stanford.EDU
Stanford University, California

Abstract

Superconductors have a profound impact and hold great potential for various applications. Despite extensive research efforts over the past century, practically valuable room-temperature superconductors remain elusive. The widely accepted Bardeen-Cooper-Schrieffer theory for superconductivity faces challenges, as it cannot adequately explain recently discovered high-temperature superconductors. The underlying issue may stem from the traditional model for electrical resistance, which incorrectly attributes resistance to particle collisions during electron flow in conductors. This study proposes an alternative theory. Similar to shear resistance states of matter, such as solids and fluids, there are also **electrical resistance states**: insulating, conducting, and superconducting states of matter at different pressures and temperatures. The electrical resistance state of matter is primarily determined by the **electron tunnel**, which develops in certain bonding between molecules at close distances. Within an electron tunnel, an electron can move between molecules without additional energy. If all electrons are below the electron tunnel, such as in conductors, they cannot flow freely between molecules, and energy is required to elevate them to the electron tunnel to create currents, which is the cause of electrical resistance. Resistivity is reduced at smaller molecular distances and the gaps between the electron tunnel and valence electrons. In a superconductor, the gap is so small that some valence electrons are within the electron tunnel, resulting in zero resistivity. There is no electron tunnel in insulators. Molecular distances can be decreased by increasing the pressure, which explains the negative correspondence between resistivity and pressure and why many high-temperature superconductors are obtained at high pressures. Molecular distances can also be reduced by lowering the temperature, as it creates an equivalent effect of increasing the pressure. For a superconductor, instead of a single critical temperature, multiple critical temperatures can be achieved at different pressures. Each critical temperature represents a point at a different pressure on the phase transition boundary of superconductivity. The proposed theory unifies the conducting mechanisms in insulators, conductors, and superconductors, providing a single framework for understanding the properties of both low and high-temperature superconductivity. Consequently, it offers promising hints and guidelines that will accelerate the search for room-temperature superconductors.

Introduction

Discovered in 1911, superconductivity has been a popular research field due to its extraordinary properties and promising applications.[1-2] Cooper pairs of electrons bound together by electron-phonon coupling at low temperatures were proposed to be responsible for superconductivity in Bardeen-Cooper-Schrieffer (BCS) theory.[3] However, since 1986, more and more superconductors have been obtained at higher and higher temperatures, substantially above the theoretical maximum prediction based on BCS theory.[4-8] Most high-temperature superconductors are obtained under high pressures, and BCS theory cannot account for the positive effect of pressure on superconductivity. In addition, BCS theory is inadequate in explaining many observed phenomena in superconductors, such as the Meissner effect.[9]

The traditional model for electrical resistance is also facing similar challenges. Most modern theories can be traced back to the Drude model, which suggests that electrical resistance is caused by collisions between flowing electrons and the crystal lattice in conductors.[10] According to the model, each collision scatters the electron flow and dissipates some of the electron's energy, resulting in electrical resistance. If the Drude model were accurate, one would expect high-density materials to be more resistive. As confining pressure increases and molecules are packed more densely in conductors, the chances of collisions increase, resulting in higher resistivity. But observations indicate just the opposite.[11-13]

The failure of theories on conductivity and superconductivity at high pressures may not be coincidental but a consequence of the implicit assumption in the collision model for electrical resistivity. These assumptions can mislead researchers and hinder both theoretical and practical progress in the field, particularly in the search for room-temperature superconductors. Rather than treating conductivity and superconductivity as distinct phenomena, we believe they should share the same physical mechanism.

Following Occam's razor principle, we propose a simple theory with the introduction of a new concept for the electron tunnel, which develops between molecules at close distances. The distances between molecules are dynamically determined by pressure and temperature, which in turn affect the resistivity of conductors. Therefore, electrical resistivity correlates with pressure and temperature. The concept of the electron tunnel is explored using mathematical models of single-atom molecules, providing valuable insights into superconductivity.

In the following sections, we will introduce fundamental concepts that construct our theory, while addressing some misconceptions that have hindered progress in this field. Utilizing the crystal structure of simple molecules, we will establish mathematical models based on the theory that help explain the properties and observations of both conductors and superconductors. The models further predict that superconductivity is an ordinary state for matter and is common at high pressures. Moreover, the different states of matter - superconducting, conducting, and

insulating - are determined by pressure and temperature. The theory and models not only offer insight into the understanding of electrical resistivity but also provide guidelines for overcoming challenges in the search for room-temperature superconductors.

Electron Tunnel

Electron tunnel refers to the network of electron paths between molecules in a conductor where electrons can flow along the paths at the same energy level, resulting in currents. An electron, with an energy level below the electron tunnel, remains confined within its orbital inside the individual molecule, and it cannot produce any current. To create currents in a conductor, electrons must be at a sufficient energy level to move through the electron tunnel. So, the space in a conductor is divided into two different types of regions: the network of electron tunnels and isolated cells around individual molecules, somewhat like cement and pebbles in a piece of concrete. An electron tunnel does not always appear in all materials. It is necessary for conductors, but absent in insulators. A superconductor is a special conductor with valence orbitals intersecting the electron tunnel. Therefore, the valence electrons move naturally in the electron tunnel without the need for energy to elevate them to the electron tunnel.

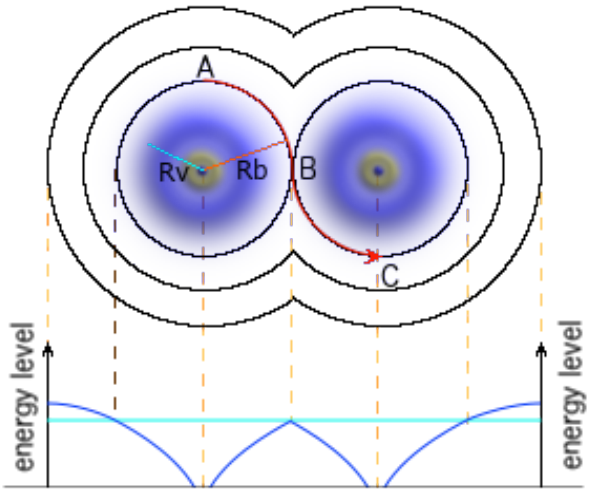

Figure 1. The electron tunnel is illustrated using a simple model involving a hydrogen atom adjacent to a proton. In this model, the valence electron is attracted not only to its nucleus but also to the neighboring proton. It demonstrates a possible attraction between molecules, which can later be extended to more realistic bonding, such as metallic bonding. The electron tunnel formed between the hydrogen atom and the proton enables the electron to move from one to the other. In the diagram, the blue short stroke represents the radius of the valence orbital, denoted by R_v, while the red stroke represents the radius to the border between the hydrogen atom and the proton, denoted by R_b. The contours represent the orbitals at different energy levels intersecting with a plane passing through the centers of the protons. The blue curves in the lower part of

the figure represent the potential field of the protons as a function of the distance from the center of each proton. An electron in the electron tunnel may drift from one proton to the other through the path A-B-C.

To introduce the concept of the electron tunnel, let's start with a simple model. Place a proton next to a hydrogen atom. The electron in the hydrogen will be attracted to the proton when it is close enough. At what distance will it happen? As shown in Figure 1, R_b represents the distance from the nucleus center to the border between the proton and the hydrogen. R_v is the radius of the valence electron orbital, which may change depending on temperature.[14] Therefore, the problem becomes to find R_b such that the electron will be pulled over to the proton.

The electron cloud and orbital shape ought to undergo deformation as atoms come into proximity. This phenomenon will be explored in subsequent discussions, particularly in the context of compression bond formation. For the sake of conceptual simplicity, we are presently assuming the absence of deformation. Additionally, the introduction of an attraction coefficient will be employed to address the uneven forces between atoms or molecules. The concept of electron tunnel will be realized later with typical bonding for the interaction between molecules. In this discussion, the term valence electrons refers to the outermost electrons in an atom, regardless of the ground or excited states.

To find the answer to the question above, we need to calculate the energy required to raise the electron from its orbital r to the border, which is the total energy difference between R_b and r. The electron is attracted to its nucleus and the adjacent proton through the Coulomb force:

$$F = \frac{Kq_1q_2}{d^2} \tag{1}$$

Here, K is Coulomb's constant, and negative F represents the attraction between charges q_1 and q_2 at a distance d. The combined force F_e, for an electron on the center line between the proton and the hydrogen nucleus, is

$$F_e = KQe\left[\frac{1}{r^2} - \frac{1}{(2R_b - r)^2}\right] \tag{2}$$

Here, Q represents the charge of a proton, e is the charge of an electron, and r is the orbital radius of the electron. The potential difference E_u of the electron between the two orbitals is the work needed to move the electron against the force F_e from orbital r to R_b, which can be computed by integrating the force F_e over the distance from r to R_b:

$$E_u = -\int_r^{R_b} F_e dx \tag{3}$$

$$= -\int_{r}^{R_b} KQe\left[\frac{1}{x^2} - \frac{1}{(2R_b - x)^2}\right]dx$$
$$= -KQe\left[\frac{1}{r} + \frac{1}{2R_b - r} - \frac{2}{R_b}\right].$$

The difference in kinetic energy must also be taken into consideration. When an electron is circulating at a distance *d* from its nucleus, its centripetal force is

(4) $$F_c = ma$$
$$= m\frac{v^2}{d}.$$

Here, *m* represents the mass of the electron, *a* is the acceleration, *v* is the velocity, and F_c is balanced by the force given in equation (2), i.e., $F_c = -F_e$. Note, the negative value in equation (2) is an attractive force. The kinetic energy at the border R_b is

(5) $$E_{R_b} = \frac{1}{2}mv^2$$
$$= -\frac{KQe}{2}\left[\frac{R_b}{R_b^2} - \frac{R_b}{(2R_b - R_b)^2}\right]$$
$$= 0.$$

The kinetic energy at orbital *r* is

(6) $$E_r = \frac{1}{2}mv^2$$
$$= -\frac{KQe}{2}\left[\frac{1}{r} - \frac{r}{(2R_b - r)^2}\right].$$

The kinetic energy difference between R_b and *r* is

(7) $$E_k = E_{R_b} - E_r$$
$$= -\frac{KQe}{2}\left[\frac{r}{(2R_b - r)^2} - \frac{1}{r}\right].$$

The total energy to lift the electron from *r* to R_b will be

(8) $$E = E_k + E_u$$
$$= -\frac{KQe}{2}\left[\frac{1}{r} + \frac{2}{2R_b - r} + \frac{r}{(2R_b - r)^2} - \frac{4}{R_b}\right].$$

Equation (8) can be represented by a 3D model of a surface that shows the lifting energy on an r-R_b plane. Alternatively, given a specific value of r, it can also be represented as a 2D curve that shows the lifting energy as a function of R_b, which represents the distance between the proton and the nucleus of hydrogen. To find the answer to the problem raised earlier, we simply need to solve equation (8) at $E = 0$:

$$(9) \qquad R_b = \frac{3+\sqrt{5}}{2} r \approx 2.618r.$$

Thus, when the proton is placed at a distance of 5.236 (2x2.618) times of electron's orbital radius from the center of the hydrogen, the electron can be pulled over toward the proton without additional energy. This means that the electron may drift over to the proton at this distance. It is important to note that the electron is far below the border between the proton and the nucleus of the hydrogen.

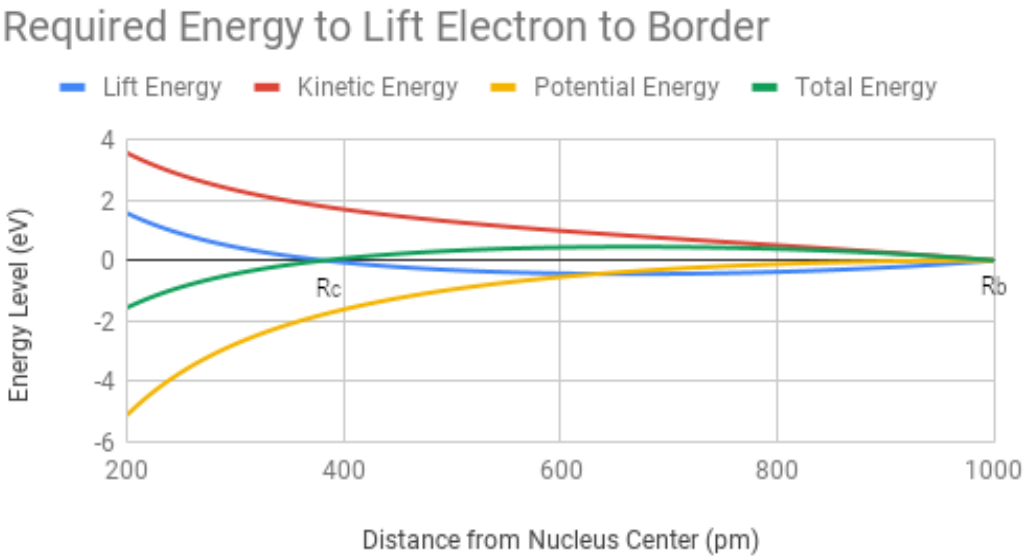

Figure 2. The curves display the required energy to raise an electron in a hydrogen atom to the border between the atom and a proton placed *2000 pm* apart. The blue curve represents the energy needed to elevate an electron to the boundary between the proton and the hydrogen nucleus as a function of the electron's orbital radius. The x-axis starts at the hydrogen center and extends to the right. R_b represents the distance to the border and is fixed at a value of *1000 pm* for the calculation of the lifting energy using equation (8). The lifting energy decreases from a positive value as the electron orbital radius increases towards the border. R_v indicates the electron radius where the required lifting energy becomes zero and turns negative beyond that point. A negative lifting energy value implies that the electron is no longer confined to its nucleus. The region with negative lifting energy values represents the electron tunnel, where the electron can move between the protons. The figure only plots the hydrogen side, with the complete

picture being symmetric and mirrored along the border. For reference, the kinetic energy (in red) based on equation (7), the potential energy (in yellow) derived from equation (3), and the total energy (in green) are also plotted. The total energy, which is the sum of the kinetic and potential energies, is the inverse of the lifting energy (in blue).

Alternatively, we can view the problem from a different perspective by looking at equation (8) as a curve that shows the energy required to raise an electron at r with a given R_b, as illustrated in Figure 2. The figure indicates that the energy required to lift an electron to the border decreases as the electron's orbital radius increases. The lifting energy reaches zero at a radius R_c:

$$R_c = \frac{3-\sqrt{5}}{2} R_b. \quad (10)$$

In other words, when an electron's orbital radius is greater than R_c, it can drift toward the proton without the need for additional energy. When a valence electron is at this radius or higher, it tends to move towards the proton, signifying that the electron is liberated from its nucleus and potentially leading to the generation of currents. Consequently, R_c is referred to as the conducting radius, which represents the condition for initiating a current between molecules in a conductor:

$$R_v \geq R_c \quad (11)$$

Here, R_v refers to the orbital radius of valence electrons. Now, the electron tunnel can be precisely defined as the region above the conducting radius between molecules. Since Figure 2 only plots the curves on the hydrogen side, the region between R_c and R_b is just half of the electron tunnel, indicated by the green curve with positive total electron energy. The other half of the electron tunnel mirrors the hydrogen side. Whenever valence electrons are in this region, they can drift between molecules, creating electrical currents.

The significance of R_c lies in its role as the dividing line between an electron's confining and unconfining orbitals. When an electron's orbital radius is smaller than R_c, it remains in an orbital confined within its atom and molecule. Electrons may experience perturbations within these orbitals. As an electron is perturbed to a higher level, its potential energy increases at the expense of kinetic energy. However, the reduction in kinetic energy is not sufficient to compensate for the rise in potential energy. This energy deficit, represented by the negative value in Figure 2's green curve, indicates that the kinetic energy decreases more than necessary to maintain the electron's speed at the higher level. As a result, the slower-moving electron tends to return to its equilibrium orbit corresponding to its energy level. Similarly, when an electron wanders to a low energy level, the reduced potential energy is converted to kinetic energy. The excess kinetic energy causes it to revert to its equilibrium orbit. Thus, without external energy, an electron below R_c is confined to its nucleus.

On the other hand, an electron with an orbital radius greater than R_c is not bound to its atom or molecule. When it is perturbed to a higher level, the excess kinetic energy, represented by the positive value of the green curve in Figure 2, propels it to an even higher level beyond the nucleus's control. The electron effectively drifts towards the proton. Upon gaining the electron, the proton transforms into a hydrogen atom, while the nucleus of the original hydrogen atom becomes a proton. Essentially, the hydrogen-proton configuration flips to a proton-hydrogen configuration. In a similar process, the electron can also drift back to the nucleus of the initial hydrogen if there are no other protons nearby. The electron essentially becomes a shared electron between the two protons, akin to a covalent bond. Indeed, a covalent bond forms through a similar interaction; however, there is a notable difference. In an H_2 molecule's covalent bond, the two electrons are bound so tightly to their nuclei that they cannot escape the molecule to generate currents.

In contrast to the tightly bound electrons in the covalent bond of H_2 molecules, the electron in the hydrogen-proton model is not restricted if there are other protons nearby. Indeed, to generate currents, a single electron tunnel between just two molecules is insufficient; a connected electron tunnel between surrounding molecules is necessary. Consider a crystal composed of half protons and hydrogen atoms. The electron tunnels between these particles can be interconnected, creating a continuous pathway for electron flow. An electron with an energy level within the electron tunnel can drift along the isoenergy level between different hydrogen atoms and protons, rather than being confined to any individual nucleus. The movement of electrons in the electron tunnel produces electrical currents. The significance of the electron tunnel lies in its function as a superhighway, facilitating the smooth flow of electrons between particles.

Keep in mind that in an isolated hydrogen atom, the total electron energy is negative and approaches zero at infinity without the presence of a nearby proton. The positive energy depicted in Figure 2 is due to the influence of the nearby proton, which implies that the electron tunnel is a result of the proton's impact. The electron tunnel enables electrons to move between molecules, generating currents. In superconductors, some valence electrons are within the electron tunnel, and their movement creates currents without the need for additional energy. In contrast, normal conductors do not have electrons in the electron tunnel, so energy is required to elevate electrons into the electron tunnel and create currents. Insulators lack an electron tunnel altogether. As a result, without a stable pathway for electron flow, energized electrons in insulators are typically pushed to the edge, forming static electrons.

The interaction discussed so far is based on a hydrogen atom and an adjacent proton. Likewise, a molecule with an electron hole can produce a similar effect to a proton, forming an electron tunnel with a neighboring molecule. Is it possible for this type of interaction to occur between two ordinary molecules? The answer is yes. This model will be extended later and generalized to accommodate realistic interactions through typical molecular bonding at various strengths.

Compression Bonding

Before discussing the generic model of the electron tunnel, let's look into a particular bonding example, i.e., **compression bonding**, that can facilitate the development of the electron tunnel. Unlike in a single isolated atom, the electrons of atoms/molecules in a conductor are influenced by the electrical fields of adjacent molecules, causing the electrons to adjust their clouds and redistribute unevenly. This induces local fields, such as London dispersions,[15] resulting in different forces and bondings between molecules. Compression bonding is a novel type of intermolecular attraction predicted in a study of superfluidity.[16]

The London dispersion creates attractive forces between normal helium molecules, resulting in a viscous helium fluid at low temperatures. As helium electrons retreat to lower orbitals at low temperatures, the London dispersion is weakened and eventually destroyed, as shown in Figure 3A. Without intermolecular attraction, there is no viscosity, and helium molecules become superfluid. As a result, a superfluid is not a fluid, but rather a collection of individual molecules. In the absence of attraction between molecules, a solid cannot be obtained by cooling the superfluid further. Pressure must be applied to obtain solid helium.[17]

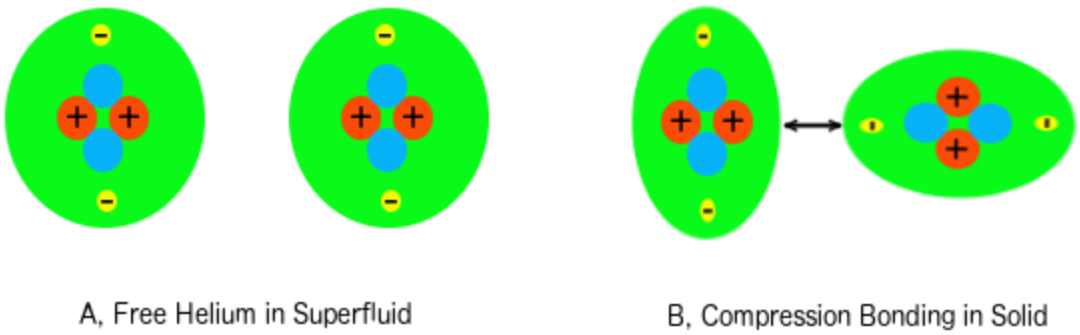

Figure 3. Compression bonding develops between helium molecules at high pressures. (A) Under normal pressures on Earth, superfluids can be obtained from liquid helium by lowering the temperature to the point where the London dispersion disappears. However, unlike most substances, solid helium cannot be produced by further reducing the temperature because there is no attraction to hold the molecules together. Pressure must be applied to obtain solid helium. (B) Under high pressures, the electron cloud of each molecule retreats and contracts along the axis connecting the two protons. This uneven distribution of electron density in different directions generates local electric fields, resulting in an attraction between molecules, referred to as compression bonding. Solid helium is formed by molecules held together through this type of bonding at high pressures.

Helium molecules become tightly compressed under pressure. Due to the electrical repulsion from adjacent molecules, the electron cloud of each molecule contracts along the axis of the two protons. This uneven electron density distribution in different directions generates local electrical fields, which are positive along the proton axis and negative at the periphery of the plane perpendicular to the axis, as shown in Figure 3B.

Attractive forces arise between the positive fields and negative fields, driving the molecules to reorient themselves to minimize their potential energy. Eventually, the molecules achieve a low potential energy arrangement, as depicted in Figure 3B, leading to compression bonding between them. Figure 3B illustrates the smallest configuration of two molecules held together by this bond. Compression bonds are responsible for holding molecules together in solid helium.[16]

In the normal state of hydrogen, shared electrons in covalent bonds are confined to the nuclei of H_2 molecules. The repulsion between H_2 molecules separates them at a considerable distance, preventing electrons from moving between different molecules. As a result, normal hydrogen is an insulator. However, under high pressure, hydrogen becomes metallic.[18-19] It is believed that the normal covalent bonds between hydrogen atoms yield to compression bonds at high pressures,[16] resulting in the formation of a continuous electron tunnel in metallic hydrogen.

Compression bonds should be prevalent in single-atom molecule substances, particularly at high pressures, as nearly everything on Earth is subject to some degree of pressure. The attractive force of compression bonding facilitates the development of electron tunnels between molecules, similar to the mechanism discussed in the previous section. The formation of electron tunnels in most conductors and superconductors composed of single-atom molecules may be related to the development of compression bonding. In a certain sense, metallic bonds can also be considered a type of compression bonding, since electron distribution in metal crystals is influenced by the fields of adjacent molecules within compact structures.

Current, Resistance, and Superconductivity

To understand superconductivity, we must first correct the misconception about electrical currents in traditional models, which has led to the incorrect collision model for resistivity. Many theories and models implicitly assume that currents are electrons flowing in the free space between molecules in conductors. Indeed, the space between molecules in conductors is never truly free for electrons. From a large distance, a molecule appears electrically neutral. However, when molecules are close together, electrical fields are induced between them, creating various intermolecular forces that hold the molecules together. Without attractive forces between molecules, there would be no solids, crystals, or conductors – but only individual molecules. The universal existence of solid materials and viscosity in fluids indicates that the attraction between molecules is prevalent. The attraction is a result of the electrical fields between them. Any electron in these fields is not free, and its movement is governed by the fields.

An electron with a negative charge cannot move freely in electrical fields, whether it is within an atom/molecule or between molecules in conductors. Within an atom of a molecule, an electron is confined to its orbital corresponding to its energy level. Although there may be some perturbations, an electron cannot change its orbital without energy exchange with its environment, such as emitting or absorbing photons. An atomic electron orbital is created by the electrical field of the nucleus. An electron tunnel may be perceived as a special orbital or an electron path created between multiple molecules. To move in this zone, an electron must also have a corresponding energy level. Below this level, electrons are confined within their molecules. Above this level, electrons are effectively out of the control of individual molecules and therefore can flow between different molecules, resulting in currents.

An electron may drift from one atom to the next atom in a different molecule along the electron tunnel, creating the current in a conductor. This process is called **electrodrift**. During an electrodrift, the energy level of the electron does not change. Before an electron can drift into the electron tunnel in a conductor, it needs to absorb additional energy to excite into the electron tunnel. An electron (or orbital) hole may be created after an electron drifts to the next molecule. The hole is unstable and may soon be filled by an electron from another molecule. A series of electron-hole drifts results in a current similar to the flow of a cation. After its electron hole is occupied by another electron, the drifted electron becomes orphaned. It may continue to drift along the electron tunnel between molecules, creating a current of negative charge. The orphan electron may eventually flow into an electron hole. Hence, currents are the flows of both negative and positive charges.

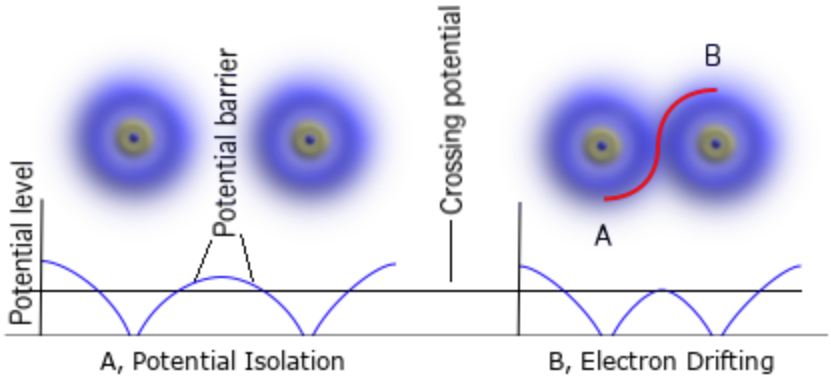

Figure 4. Molecular distance is the primary difference between normal conductors and superconductors. As shown in the figure, each atomic nucleus generates an electrical field that attracts the electrons surrounding it. The energy level is represented by the blue curves in the lower part of the figure. The horizontal direction indicates the distance between the atoms, and the vertical direction represents the energy level. (A) At a considerable distance, there is an energy barrier between the atoms, separating the electrons. To enable electrodrift, energy is needed to elevate electrons to the electron tunnel along the border, which is the cause of electrical resistance in conductors. (B) By increasing pressure or decreasing temperature, the distance between the atoms can be reduced, pushing the electron tunnel lower to overlap the valence orbitals. When the

valence orbital lies within the electron tunnel, electrons can drift from one atom to another without the need for lifting energy, as illustrated by the red path A-B, leading to superconductivity.

There is normally no current in conductors, even though there are electron tunnels in them. Electrons are typically confined in their atomic orbital within a molecule and cannot move between molecules, as illustrated in Figure 4A. The space between molecules is not just empty but separated by potential barriers. For an electron to move from one molecule to the next, it has to overcome these energy barriers. To create a current, an electron must be elevated to the electron tunnel. An electron may be raised to the electron tunnel in an electrical field, such as an applied voltage, or a magnetic field, such as in an electrical generator.

The lifting energy is the work against the Coulombic force between the electron and its nucleus. After gaining additional energy, the lifted electron can flow in the electron tunnel between molecules, forming a current. The absorbed energy will eventually dissipate through emitting photons, i.e., radiating heat, as the electron retreats into an electron hole. Therefore, the cause of electrical resistance is due to the Coulombic force, not the collisions between particles as conceived in the traditional model.

Unlike in a normal conductor, where the electron tunnel is located above the valence orbital, in a superconductor, the valence orbital intersects the electron tunnel. This intersection allows for electrodrift to occur naturally in the electron tunnel without the need for additional lifting energy, as shown in Figure 4B, resulting in currents without resistance – the superconductivity. The electron tunnel can be pushed lower to overlap valence orbitals by increasing pressure or decreasing temperature, which will be discussed in more detail later.

It appears that superconductivity requires no energy at all, which is not true. At a temperature above *0 K*, electrons both absorb and emit energy from the environment, resulting in a net energy exchange of zero. The valence electrons in a conductor should be in a statistical equilibrium at an energy level above the ground orbitals. From the perspective of an electron, the total energy E_t needed to elevate from the ground orbital to the electron tunnel can be broken down into two components: E_v the energy required to elevate from the ground orbital to the valence orbital, and E_c the energy required to raise from the valence orbital to the electron tunnel:

$$E_t = E_v + E_c. \quad (12)$$

In equilibrium, E_v is maintained or supplied by the surrounding environment. E_c is the only additional energy needed to lift the electron for electrodrift, which represents the work required to create currents in a conductor and is the cause of electrical resistance. When the valence orbital is within the electron tunnel, as is the case in a superconductor, the lifting energy is unnecessary for electrodrift, i.e., $E_c = 0$. Thus, equation (12) can be simplified to

(13) $E_t = E_v$

This indicates that the lifting energy for electrons in superconductors is constantly supplied by the environment. At equilibrium, the energy exchange between a superconductor and its environment is balanced. From the environment's perspective, its energy loss is compensated by the energy radiated from the superconductors. The entire system is still energy-conservative. Theoretically, when a superconductor and its environment are at *0 K*, there is no energy exchange between the superconductor and the environment. The valence orbitals are at the ground level. The equations (12) and (13) still hold because

(14) $E_t = E_v = E_c = 0.$

Generic Model for Electron Tunnel

The electron tunnel concept introduced so far is based on a simplified model involving a hydrogen atom and a proton. In reality, electron tunnels can be developed by the interactions between molecules through various bonds. Outer shells of the electron cloud of atoms in a molecule are typically distributed unevenly, resulting in various intermolecular forces. Intermolecular attractions, or bonds, hold molecules at a close distance, which is crucial to the development of electron tunnels, enabling electrons to flow between molecules. It is important to note that molecular bonds, such as covalent bonds, allow electrons to move between atoms within individual molecules but do not facilitate electron movement between different molecules necessary for the current generation.

To learn the influence of bonding intensity on electron tunnels and to identify the boundary between the electron tunnel and molecule cells in a conductor, the concept of attraction coefficient is introduced, denoted by the symbol *c*. This coefficient models the intensity of the attraction of an electron by an adjacent molecule, such as that due to a metallic bond. Assuming that an electron is attracted by its nucleus with an equivalent charge *Q*, taking into account other electrons in the same molecule, the attraction to the electron by an adjacent molecule in the crystal lattice can be modeled as if it were from a charge of *cQ*. The value of c is typically between *0* and *1*. Hence, the force exerted on the electron by both molecules can be determined, even when considering all the molecules in the crystal. Consequently, the energy level of the electron and the potential fields between molecules can be calculated at any given location within the lattice structure.

To estimate the energy required to lift an electron from orbital *r* to the border R_b, we need to consider all the forces exerted on a valence electron between two molecules. By incorporating the coefficient *c*, we can refine equation (2) in the previous model to

(15) $$F_e = KQe\left[\frac{1}{r^2} - \frac{c}{(2R_b - r)^2}\right]$$

Here, *c* <= *1,* denoting the attraction coefficient for the next molecule. When *c* = *1*, the model represents two molecules with an equivalent attraction to an electron between them, such as a normal molecule (or hydrogen atom) next to a molecule with an electron hole (or proton), resulting in the same model as in equation (2). When *c* = *0*, it simulates a single molecule without adjacent molecules. With a value of *c* between *0* and *1*, it models various bonding strengths between molecules. Indeed, it will become clear later that the value of *c* signifies the difference between insulators, conductors, and superconductors. For values of *c* that are small enough, e.g., *c* < ⅛, the model represents the fields in insulators. Otherwise, such as between *¼* and *1*, it simulates the attraction fields between molecules in conductors and superconductors.

Next, let's also include the forces from two molecules located along the center line on farther sides of the two initial molecules in the crystal grid of a conductor. Equation (15) needs to be adjusted accordingly:

(16) $$F_e = KQe\left[\frac{1}{r^2} - \frac{c}{\left(2R_b - r\right)^2} + \frac{c}{\left(2R_b + r\right)^2} - \frac{1}{\left(4R_b - r\right)^2}\right].$$

Now, consider the forces from two molecules further down the line, so on and so forth. The force exerted on the electron from all the molecules along the center line is

(17) $$F_e = KQe\sum_{i=0}^{N}\left[\frac{c^{i\%2}}{\left(2iR_b + r\right)^2} - \frac{c^{(i+1)\%2}}{\left(2(i+1)R_b - r\right)^2}\right]$$

Here, the symbol *%* represents the modulo operation or MOD, and *N* should be a large number determined by the size of the conductor. In a crystal of a conductor, there are also molecules surrounding the center line along the two initial molecules. If there is an infinite number of molecules, their influence will cancel due to the symmetry structure along the center line. Considering a sizable conductor, it is reasonable to assume the effect of surrounding molecules will cancel each other out. Therefore, the molecules along the line are all we need to be concerned about, and equation (17) should be sufficient for our model. The model in equation (17) can be further generalized for large molecules by interpreting *Q* to be equivalent positive charges that exert the Coulombic force on the electron.

The potential energy difference, E_u, or the energy needed to move the electron along the center line from orbital *r* to the border R_b between two molecules, can be computed by integrating over the force provided in equation (17) from *r* to R_b:

$$(18)\quad E_u = -\int_r^{R_b} KQe \sum_{i=0}^{N}\left[\frac{c^{i\%2}}{\left(2iR_b+x\right)^2} - \frac{c^{(i+1)\%2}}{\left(2(i+1)R_b-x\right)^2}\right]dx.$$
$$= -KQe\sum_{i=0}^{N}\left[\frac{-c^{i\%2}}{2iR_b+x} + \frac{-c^{(i+1)\%2}}{2(i+1)R_b-x}\right]_r^{R_b}$$
$$= -\frac{KQe}{2}\sum_{i=0}^{N} D_u(i)$$

where

$$(19)\quad D_u(i) = \frac{2c^{i\%2}}{2iR_b+r} + \frac{2c^{(i+1)\%2}}{2(i+1)R_b-r} - \frac{2c^{i\%2}+2c^{(i+1)\%2}}{2iR_b+R_b}.$$

The kinetic energy for an electron at a distance *r* from the nucleus along the center line can be calculated from equation (17):

$$(20)\quad E_r = -\frac{KQe}{2}\sum_{i=0}^{N}\left[\frac{rc^{i\%2}}{\left(2iR_b+r\right)^2} - \frac{rc^{(i+1)\%2}}{\left(2(i+1)R_b-r\right)^2}\right].$$

The kinetic energy difference between R_b and *r* is

$$(21)\quad E_k = -\frac{KQe}{2}\sum_{i=0}^{N} D_k(i)$$

where

$$(22)\quad D_k(i) = \frac{rc^{(i+1)\%2}}{\left(2(i+1)R_b-r\right)^2} - \frac{(c^{(i+1)\%2}-c^{i\%2})R_b}{\left(2iR_b+R_b\right)^2} - \frac{rc^{i\%2}}{\left(2iR_b+r\right)^2}.$$

The total energy, E_c, to elevate an electron from orbital r to the border between two molecules is the sum of the potential energy and kinetic energy estimated in equations (18) and (21), respectively:

$$(23)\quad E_c = E_u + E_k$$
$$= -\frac{KQe}{2}\sum_{i=0}^{N}\left[D_u(i) + D_k(i)\right].$$

We will show that the summation term in this equation is proportional to the electrical resistivity of a conductor. So, let's name it the **resisting distance**:

(24) $$D = \sum_{i=0}^{N} \left[D_u(i) + D_k(i) \right].$$

Equation (23) estimates the energy needed to elevate an electron from orbital r to the border R_b along the center line between two molecules. For valence electrons, $r = R_v$. Therefore, the same equation can be used to estimate the required energy to lift valence electrons to create currents. In the next section, we will establish the relationship between the lifting energy and the resistivity of a conductor, which provides a theoretical base for the correlation between the resistivity and molecular distance of a conductor.

Resistivity and Resisting Distance

The electrical resistance of a conductor is described by Ohm's law, which is an empirical relation obtained from experiments. What are the fundamental microscopic properties of a conductor that give rise to this law? In this section, we will explore the relationship between a conductor's resistivity and its macroscopic properties related to the resisting distance.

The energy estimated in equation (23) represents the minimum energy required to elevate a valence electron to the electron tunnel for creating a current. Thus, the lifting electrical potential for the electron is

(25) $$v = \frac{E_c}{e}.$$

Here, e is the charge of an electron. Assume a total voltage V is applied to the ends of a conductor with a length L and a cross-sectional area A. The voltage elevates and drives n electrons through the conductor in T seconds. Hence, V is the total potential to create the current of n electrons, which provides the potential v for driving individual electrons. At the microscopic level, the voltage V applied to the conductor by a power supply, such as a battery, is achieved through the accumulation of electrical charges at the ends of the conductor, which provides the potential to lift electrons near the ends. The lifted electrons create a potential field further into the conductor, which in turn provides the potential field to drive electrons further into the conductor, and so on. Therefore, V should be proportional to the lifting potential, v, and the number of electrons, n, and they can be related by introducing a coefficient p:

(26) $$V = pnv.$$

By definition, the current I created by V is

(27) $$I = \frac{en}{T}.$$

Combining equations (25), (26), and (27), we can find the relation between electrical resistance, R, and lifting energy, E_c, based on Ohm's law:

(28) $$R = \frac{V}{I} = \frac{pTE_c}{e^2}.$$

By definition, the resistivity of a conductor is

(29) $$\rho = R\frac{A}{L} = \frac{pATE_c}{Le^2}.$$

Replacing E_c provided in equation (23), the electrical resistivity becomes

(30) $$\rho = -\frac{pATKQ}{2Le}D.$$

Note, the speed of electrons drifting in a conductor is

(31) $$s = \frac{L}{T}$$

This is a property specific to a conductor. By introducing an equivalent coefficient u for the nuclear charge of molecules in the conductor, Q can be expressed as

(32) $$Q = -ue.$$

And equation (30) can be simplified to

(33) $$\rho = \frac{puAK}{2s}D.$$

For hydrogen, $u = 1$. As molecule size increases, the valence electrons experience less force from the nucleus and $u < 1$. By introducing a property Z, namely microscopic resistivity, which encapsulates the Coulomb's constant and microscopic properties p, u, and s specific to a conductor,

(34) $$Z = \frac{puAK}{2s}$$

The resistivity can be simplified as

(35) $$\rho = ZD$$

Here, the microscopic resistivity, Z, represents the static properties of a conductor at the microscopic level, while the resisting distance, D, encapsulates the dynamic properties: R_v, R_b,

and *c*. These dynamic properties depend on the distance between molecules and change dynamically with pressure and temperature. Therefore, the dynamic nature of a conductor's resistivity can be explored through the resisting distance.

Resisting Distance

The resisting distance, *D*, of a conductor is a common factor of the resistivity indicated in equation (35) and the lift energy for electrons to create currents in the conductor based on equation (23). The theoretical value for *D*, estimated in equation (24), is a sum of a large series. It can be easily demonstrated that the series converges even for an infinite series because each term in the series converges to zero at the rate of $O(n^{-2})$. First, each term in equation (22) for $D_k(i)$ converges at the same rate as $O(n^{-2})$. Second, we can also prove that the terms in $D_u(i)$ converge by reformulating equation (19) as follows:

$$D_u(i) = \frac{2c^{i\%2}(R_b - r)}{(2iR_b + r)(2iR_b + R_b)} + \frac{2c^{(i+1)\%2}(r - R_b)}{(2(i+1)R_b - r)(2iR_b + R_b)} \quad (36)$$

Here, each term converges at the same rate as $O(n^{-2})$. Mathematician Leonard Euler proved the convergence of the infinite series in the Basel problem, specifically, $\Sigma(1/n^2) = \pi^2/6$. As a result, each term in the series converges to $A\pi^2/6$, with *A* being a constant for particular values of *c* and R_b. Consequently, the sum of these terms converges for each tuple of *c* and R_b. This demonstrates that the model predicts a finite electrical resistance in a conductor, therefore requiring a finite amount of energy to elevate electrons to the electron tunnel for creating currents.

Analyzing the resisting distance using the series form in equation (24) is challenging. As the influence on an electron from surrounding molecules decreases rapidly with distance, i.e., at the rate of $O(n^{-2})$, the first term *d* in the series should be significant enough for the analysis:

$$d = \frac{1}{r} + \frac{2c}{2R_b - r} + \frac{rc}{(2R_b - r)^2} - \frac{3c+1}{R_b}. \quad (37)$$

Indeed, a numerical simulation indicates that the ratio of *D/d* converges rapidly and is bounded by a constant for any specific values of *c* and R_b. Thus, instead of working with the entire series of the resisting distance, we can just focus on the first term *d* of the series, which should be significant enough to provide insight into the resisting distance.

First, let us explore the effect of R_v and R_b on the resisting distance. Equation (37) allows us to estimate the resisting distance for a conductor using its microscopic properties R_v and R_b, along with the property *c*. Unfortunately, the values for *c* are currently unavailable for most materials. To work around this issue, we can assume a uniform value of $c = 0.1$ and use it to estimate the range for the resisting distances for several materials. By using the values of R_v and R_b obtained

from the ptable site, we calculated the corresponding resisting distances for these materials and compiled them in the Resisting D column of Table 1.

Name	Symbol	Border Ra	Valence Ra	Resisting D (1/m)	Resistivity (m/MS)
Silicon	Si	210	111	5.37E+13	1.00E+03
Diamond	C	170	77	8.50E+11	1.00E+01
Aluminum	Al	125	118	1.87E+07	2.62E-02
Calcium	Ca	180	174	9.00E+06	3.45E-02
Iron	Fe	140	125	2.42E+08	1.00E-01
Copper	Cu	140	138	8.41E+05	1.69E-02
Platinum	Pt	175	128	1.50E+09	1.06E-01
Gold	Au	166	144	5.81E+07	1.61E-02
Mercury	Hg	155	149	3.49E+08	1.00E+00
	c = 0.1			Radius in pm	

Table 1. Resistivities of several materials are compared with the resisting distances estimated using equation (37). The Resisting D column shows the resisting distance (m^{-1}) for different materials, which were calculated using equation (37), assuming a uniform value of $c = 0.1$. The values of R_v and R_b for each material are obtained from the ptable site. Specifically, the Valence Ra column is used for R_v, which corresponds to the covalent radius, and the Border Ra column for R_b, which is based on the Van de Waals radius or empirical radius for Ca, Fe, and Al. For simplicity, we did not include the common factor of property Z in the calculation.

The results indicate a strong correlation between the electrical resistivity (column Resistivity) and the resisting distance (column Resistive D), even when assuming a uniform value of $c = 0.1$. Notably, non-conductive materials such as silicon and diamond exhibit much higher resisting distances than metals. Theoretically, the electrical resistance of any material can be computed using equation (35) as long as accurate values are available for the microscopic properties Z, R_v, R_b, and c.

Now, let's examine the impact of the attraction coefficient, c, on resisting distance. The attraction coefficient characterizes the bonding strength of the attraction between molecules, such as compression bonding. According to equation (37), if there is no attraction field between molecules (i.e., $c = 0$) or the field is weak (e.g., $c < ⅛$), the resisting distance is always greater than zero, as indicated by the blue curve in Figure 5. This suggests the absence of an electron tunnel, which is the situation in an insulator where electrons are mostly confined within their molecules. It typically requires much more energy to free the electrons from molecules.

An electron tunnel appears in substances with high attraction coefficients (e.g., $c > ¼$). As c increases, the width of the electron tunnel also increases. For instance, when $c = ⅓$, there is a small electron tunnel, as indicated by the red curve, while at $c = 1$, there is a wider electron

tunnel, as shown by the yellow curve in Figure 5. The electron tunnel provides a stable path for electron flow in conductors.

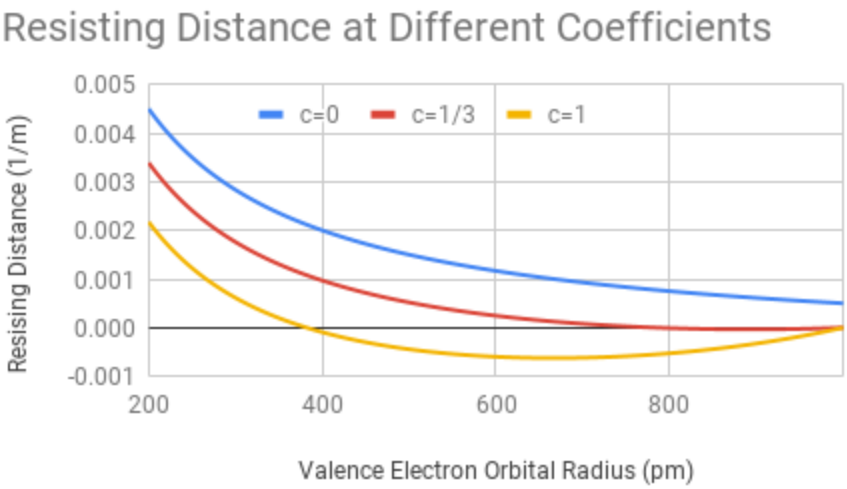

Figure 5. The resisting distances (d m^{-1}) are compared for different values of c, calculated using equation (37) with molecules *2000 pm* apart and a fixed value of R_b = *1000 pm*. The curves demonstrate that the resisting distance decreases as R_v increases towards R_b. The orbital radius at zero resisting distance is where the curves intersect the x-axis, denoted as R_c, indicating the starting point of the electron tunnel. The width of the electron tunnel increases with c. When $c = 0$, the resisting distance is always greater than zero, as indicated by the blue curve, suggesting that there is no electron tunnel, which is the case for insulators. A superconductor is a conductor with $R_v >= R_c$, indicating that valence electrons are within the electron tunnel.

In normal conductors, electrons have to be energized into the electron tunnel. The larger the gap between the valence orbital and the electron tunnel, the more energy is required to elevate the valence electron to the electron tunnel, and the higher the resistivity. This means that the resistivity of a conductor usually decreases with increasing c, as the gap between the electron tunnel and the valence orbital is typically small at wider electron tunnels. When the electron tunnel is wide enough to overlap valence orbitals, valence electrons are naturally situated and flow in the electron tunnel without lifting energy or resistance, which is the state of superconductivity.

So, let's explore the condition for superconductivity. As indicated above, in superconductors, there is no resistivity, i.e., $\rho = 0$. Using equation (37), the superconductivity condition can be expressed as

$$(38) \quad \frac{1}{R_c} + \frac{2c}{2R_b - R_c} + \frac{cR_c}{(2R_b - R_c)^2} - \frac{3c+1}{R_b} = 0.$$

Here, R_c denotes the conducting radius, which is the point where the electron orbital begins to intersect the electron tunnel. In Figure 5, this is also the point at which the resisting distance curve intersects the x-axis, and the resisting distance becomes zero, i.e., $D = 0$. Thus, whenever $R_v >= R_c$, superconductivity is achieved. By solving equation (38) for R_c at different values of c, we found:

$$(39) \quad \begin{aligned} R_c &\approx 0.382R_b \quad when \quad c = 1 \\ &\approx 0.621R_b \quad when \quad c = \frac{1}{2} \\ &\approx 0.785R_b \quad when \quad c = \frac{1}{3}. \end{aligned}$$

The results imply that electrodrift may occur well below the border between molecules. With $c = ½$, for instance, valence electrons will be in the electron tunnel when $R_v > 0.621R_b$, and superconductivity will occur. The reason why electrodrift can take place below the border is due to the pulling effect of the attraction from adjacent molecules. This typically occurs in a bonding field, such as compression bonding.

The total potential and kinetic energy of an electron is conservative. An electron's orbital results from the perturbation of the electron along its path. As the electron moves to a higher level, its potential energy increases at the expense of its kinetic energy. The decrease in kinetic energy causes the electron to move more slowly and fall back to a lower level, thus decreasing its potential energy while increasing its kinetic energy. As a result, the electron oscillates around its equilibrium orbit in an overall stable orbital. However, beyond the conducting radius, the attraction field from the adjacent molecule becomes significant enough and can pull the electron off its confining orbital, resulting in an electrodrift between molecules.

When $c = 1$, a valence electron experiences an attraction from an adjacent molecule that is equivalent to the force it would experience from its parent molecule at the same distance. This scenario may occur when there is an electron hole in the adjacent molecule, which can happen after an electron jump and electrodrift. This implies that if a valence electron is situated next to a molecule with an electron hole, it can drift to the next molecule without requiring additional lifting energy when the valence electron is at an orbital above 0.382 of the distance to the border. In a typical bond between molecules, $c < 1$. For example, when $c = ⅓$, the electrodrift will occur at 0.785 of the border distance.

Therefore, the difference between insulators, conductors, and superconductors can be quantitatively identified based on the solution to equation (38). For a given material with a specific value of c, if the equation has no real solution, then the material is an insulator. If there exist real solutions, the material is either a conductor or a superconductor, depending on whether $R_v < R_c$ or $R_v >= R_c$, respectively. This approach provides a method to distinguish between these three types of materials based on their microscopic properties and the bonding strength between molecules.

Resistivity Dynamics

Through the investigation of the effects of c, R_v, and R_b on the resisting distance using the established model, we have uncovered the microscopic conditions necessary for superconductivity (i.e., $R_v >= R_c$) and the differences between various electrical resistance states of a matter based on the appearance of the electron tunnel. With this understanding, we can now explore the dynamic aspects of electrical resistivity, which are primarily influenced by changes in pressure and temperature.

Increasing pressure can reduce the distance between molecules, leading to the induction of various intermolecular fields and forces, such as the development of compression bonding. This process increases the attraction coefficient c. As c increases, the electron tunnel expands, as demonstrated in the previous section. The widening of electron tunnels typically decreases the gap between valence orbitals and electron tunnels, reducing the resisting distance and ultimately the electrical resistivity. These findings highlight the intricate relationship between pressure and resistivity. This explains why resistivity decreases with increasing pressure and some insulators become superconductors at high pressures.

Another factor influencing the resisting distance, D, is the border radius, R_b, which is also sensitive to pressure. R_b decreases monotonically as pressure increases. When R_v is kept constant, reducing R_b effectively increases R_v and typically decreases the gap between R_v and R_c, reducing D and resistivity. As illustrated in Figure 5, D diminishes as R_v increases for all selected values of c. Consequently, D shrinks with a decrease in R_b, leading to a reduction in resistivity, which reinforces the effect of the attraction coefficient on increasing pressure.

The final variable affecting the resisting distance, D, is the electron's orbital radius, r, or more specifically, the valence orbital radius, R_v, which is more sensitive to temperature. At higher temperatures, valence electrons become excited and move to higher orbitals, causing R_v to increase. When there is enough pressure to confine the distance between molecules, increasing R_v will decrease the gap between R_v and R_c, and therefore reduce D and resistivity, as demonstrated in Figure 5.

However, without confining pressure, increased repulsion from electrons in adjacent molecules pushes the molecules apart, leading to an increase in R_b. Under normal pressure on Earth, which is relatively weak compared to the repulsion between molecules, R_b typically increases faster than R_v, which in turn expands the gap between R_v and R_c. Consequently, D increases, as illustrated in Figure 6. This explains why traditional superconductors, achieved at low temperatures, are usually destroyed at high temperatures.

At low temperatures, electrons tend to retreat to lower orbitals, and repulsion between molecules weakens. This causes the normal pressure on Earth to become more significant,

creating an equivalent compression effect with respect to the constant surrounding pressure on Earth. As R_b decreases, R_c is pushed lower faster than the reduction of R_v, eventually placing the valence electrons in the electron tunnel. This results in conventional superconductors at low temperatures, as shown in Figure 6A. As the temperature increases, the process reverses, and electron tunnels are pushed up and out of the valence orbitals, destroying superconductivity, as shown in Figure 6B. In other words, the effect of lowering temperature is somewhat equivalent to increasing pressure.

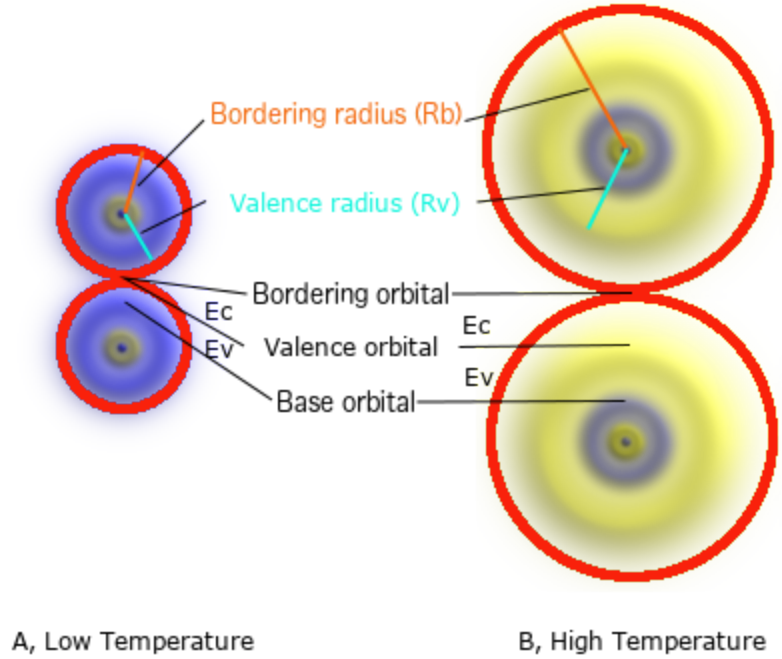

Figure 6. The transition of the electrical resistance state is illustrated when the conventional superconductivity is destroyed at high temperatures. (A) Conventional superconductors are typically observed at low temperatures, where molecules are so close that valence electrons are within the electron tunnel, allowing electrodrift to occur freely without the need for lifting. (B) As the temperature increases, molecules are pushed apart due to the increasing intermolecular repulsion caused by excited electrons at higher orbitals. The conducting radius rises faster than the valence orbital. Eventually, the valence orbitals fall below the electron tunnel, destroying superconductivity.

In summary, the distance between molecules is dynamically determined by pressure and temperature, where pressure plays a particularly crucial role, and temperature change creates an equivalent pressure effect. Molecular distance determines the border radius and also influences the induction of the attraction field between molecules, such as the field in compression bonding. The attraction field or force between molecules controls the attraction coefficient, which is the primary factor that determines the electron tunnel and the electrical resistivity. Therefore, the resistance state of matter is dynamically influenced by both pressure and temperature.

Electrical Resistance State

We have discussed the dynamic nature of resistivity, which varies with changing pressure and temperature. It is more important to note that superconducting, conducting, and insulating materials can transition from one to another. Insulators can become superconductors with sufficient pressure, as observed in many high-temperature superconductors. Mercury, which is a conductor at normal temperatures, transitions to a superconductor at low temperatures. Just like states of matter can be classified based on their shear resistance, such as solids and fluids, they may also be categorized based on their electrical resistance. This transition of electrical states depends on the presence and width of the electron tunnel between molecules. The electron tunnel is determined by the attraction coefficient between molecules, which in turn is affected by pressure and temperature. Therefore, the electrical resistance state boundaries can also be presented on the same state diagram as shear resistance states.

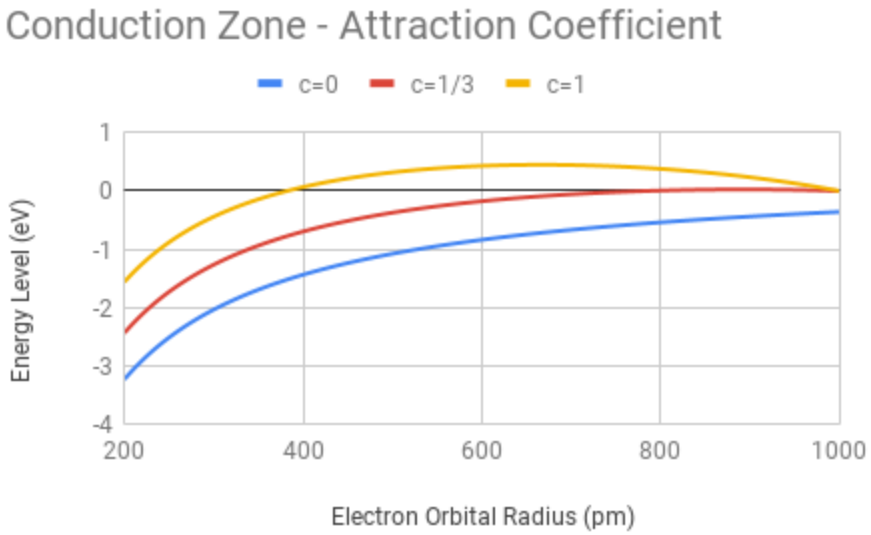

Figure 7. The relationship between electron tunnel width and attraction coefficient is illustrated using a model in which a molecule is located *2000 pm* away from the next molecule with a bonding strength represented by an attraction coefficient *c*. Each curve represents the energy levels of an electron as a function of the electron orbital radius for a specific attraction coefficient, calculated using equation (23) with a fixed R_b of *1000 pm*. The x-axis starts at the molecule center and extends toward the adjacent molecule to the right. The figure plots only the sections from *200 pm* away from the molecule center to the border with the next molecule along the center line between the two molecules. The electron energy level increases as the orbital radius increases from the molecule's center to the border. When $c = 1$, the energy level increases from negative values, turning positive at a radius of *382 pm*, as illustrated by the yellow curve. A positive energy level indicates the electron is no longer confined by its nucleus and can flow between molecules. Thus, the region of the positive energy level represents the

electron tunnel. Note that with $c = 1$, the curve simulates the attraction from the next molecule with an electron hole, creating the widest electron tunnel. When $c = ⅓$, the energy level turns positive at a radius of *785 pm*, resulting in a smaller electron tunnel, as shown by the red curve. When $c = 0$, the energy level remains entirely negative, indicating no electron tunnel, as illustrated by the blue curve.

Increasing pressure reduces the distance between molecules, which also induces bonding between them. Hence, pressure determines the electrical resistance state of matter. As illustrated in Figure 7, the width of the electron tunnel decreases with increasing attraction coefficients, which may represent the electrical resistance states of a substance transitioning from superconducting to conducting or insulating at different pressures. Attraction coefficients model the attractive forces of molecular bonding. Attraction, such as compression bonding, is induced between molecules at small distances. Indeed, most intermolecular bonds form at close distances. Molecules naturally repel each other due to their outer electrons. Confining pressure restricts them together and reduces the distance between them. At close distances, electrons are influenced by adjacent molecules and redistribute, which induces different fields and creates attraction and bonding between molecules. As discussed earlier, decreasing temperature has an equivalent effect to increasing pressure. This is why the electrical resistance state of matter is a function of both pressure and temperature.

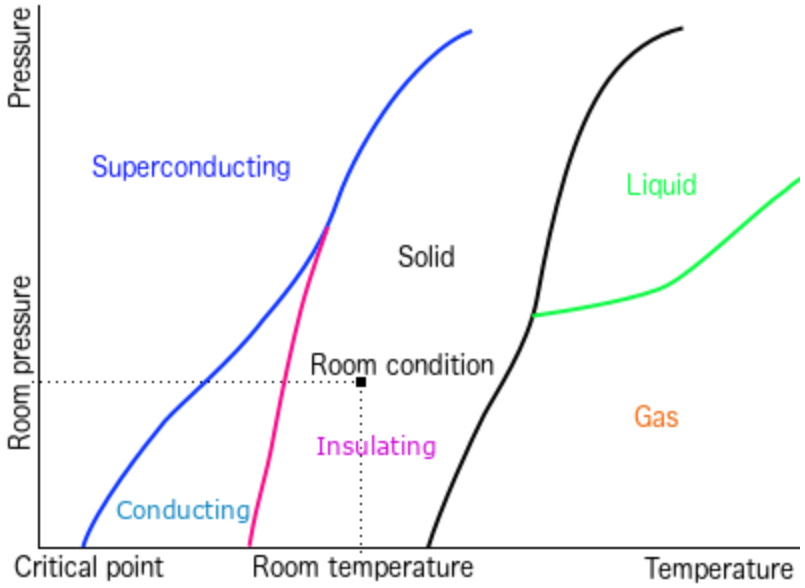

Figure 8. An illustrative electrical resistance state diagram depicts superconducting, conducting, and insulating transition boundaries, besides the transition boundaries of conventional shearing resistance states. The transition boundary for superconductivity is the curve connecting all the tuples of critical temperatures and pressures of the superconductor. It typically resides on the low-temperature and high-pressure side of room conditions, explaining why room-temperature superconductors are less common due to unfavorable conditions on Earth. An insulating transition boundary of a substance

may intersect with the superconducting transition boundary when there is a direct transition from an insulator to a superconductor.

Hence, just as solids and fluids represent different shear resistance states of matter at different temperatures and pressures, substances can also exhibit different **electrical resistance states**, including superconducting, conducting, and insulating states. The electrical resistance state of a substance can transition from one to the other with variations in pressure and temperature. In the case of a conventional superconductor, the critical point is merely a transition temperature observed at normal pressure on Earth. Multiple critical temperatures may be observable at different pressures for the same superconductor. Each of the critical temperatures represents a point on the superconducting transition boundary at a specific pressure. All the critical points connect to form a superconducting transition boundary in a state diagram, as illustrated by the blue curve in Figure 8.

The transition boundary can also be theoretically determined from the model. Equation (35) can be interpreted as a 3D surface of resistivity over a domain of pressure and temperature. The transition boundary is the intersection between the surface and the zero resistivity plane, i.e., $\rho = 0$, which is represented by equation (38). The solution to equation (38) can be expressed in the following form:

$$R_c = f(c, R_b) \tag{40}$$

Here, each term of $R_c(t,p)$, $R_b(t,p)$, and $c(t,p)$ is a function of temperature t and pressure p. Equation (38) may not have a real solution when the intermolecular attraction is weak, e.g., $c <$ ⅛, which corresponds to the insulating state. However, when there is a real solution with a given c, such as in the form of equation (40), all the points of (t,p) define the transition boundary between superconducting and conducting states. Any point (t,p) with $R_v(t,p) > R_c(t,p)$ must be on the superconducting side, while points with $R_v(t,p) < R_c(t,p)$ are on the conducting side. For instance, a real solution of equation (40) at $c = 1$ is

$$\begin{aligned} R_c &= \tfrac{3-\sqrt{5}}{2} R_b \\ &\approx 0.382 R_b. \end{aligned} \tag{41}$$

In this case, the material will exhibit superconductivity whenever the combination of pressure and temperature moves the valence electrons to an orbital at 0.382 or greater of the distance to the border. Otherwise, it will behave as a conductor.

Since the decreasing temperature has an effect equivalent to increasing pressure, the curve must generally trend from low pressure and temperature toward high pressure and temperature, as illustrated by the blue curve in Figure 8. For conductors, the difference between $R_v(t,p)$ and

$R_c(t,p)$ correlates to resistivity. A large difference between $R_v(t,p)$ and $R_c(t,p)$ indicates that more energy is required to elevate valence electrons to the electron tunnel.

The electron tunnel disappears when the attraction coefficient decreases to a value of about *0.225*. Therefore, all the points *(t,p)* on the temperature-pressure plane where $c(t,p) = 0.225$ define the insulating boundary, as illustrated by the purple curve in Figure 8. This boundary must be on the high-temperature side of the superconducting boundary. There may be overlaps between superconducting and insulating boundaries, which form the boundary of transition from insulating to superconducting directly. A triple transition point may also exist in an electrical resistance state diagram.

The phase transition boundaries in the diagram illustrate the theoretical predictions based on the model. The actual transition boundaries for some materials may not be smooth curves due to the discrete nature of atomic orbitals. Some superconducting phase transition boundaries may exhibit stepwise behavior for materials with large energy gaps between different orbitals. Based on this theory, superconductivity is common at high pressures. If superconductivity is an ordinary state of matter as predicted, why are superconductors not as common as conductors? The reason is that the typical combinations of temperatures and pressures on Earth are not favorable for superconductivity.

Meissner Effect

A superconductor is not only a perfect conductor, but it is also significant because of the Meissner effect, a phenomenon where an external magnetic field is expelled from the superconductor during the transition to the superconducting state.[9] It is important to note that a magnetic field created by induction requires a change in magnetic flux, as described by Faraday's law of induction.[20-21] The Meissner effect is observed during the transition to the superconducting state in the presence of an existing magnetic field, where there is no change in magnetic flux.

Electrons in any material usually excite and de-excite simultaneously. In normal conductors, there is no current because the valence electrons are below the electron tunnel. After the transition to the superconducting state, the electron tunnel overlaps the valence orbitals. The valence electrons can flow between molecules without the need for lifting, resulting in electrodrifts. Each electrodrift creates an electron hole and an orphan electron. The drift of the electron hole results in a current of positive charge, while the drift of the orphan electron produces a current of negative charge. Without an external magnetic field, the currents would not create observable magnetic fields because the fields resulting from the random currents cancel each other out locally. In the presence of an external magnetic field, the directions of the currents are deflected by the Lorentz force:

$$F = q(E + v \times B) \quad (42)$$

Here, q represents the electrical charge, *v* represents the velocity of the charge, *B* represents the magnetic field, *E* represents the electric field, and *F* represents the force exerted on the charge.[22-24] When observing along the direction of the applied magnetic field, a moving electron is deflected by the Lorentz force and begins to circulate in a clockwise direction, while the drift of an electron hole circulates in an anti-clockwise direction. As a result, the moving charges circulate in the superconductor, generating magnetic fields that counteract the applied magnetic field inside the superconductor and reinforce the applied field outside of the superconductor. The net result creates the appearance that the applied field is expelled from the superconductor.

The relationship between the Meissner effect and Faraday's law of induction becomes evident at the microscopic level. In both mechanisms, the generated magnetic fields result from the flow of charges being deflected by the Lorentz force. The primary difference lies in the fact that random currents naturally exist in a superconductor due to valence electrons being present within the electron tunnel. In contrast, there are typically no currents in a conductor, and electrons must be raised to the electron tunnel to create currents. Induction also occurs in a superconductor when the external field changes. As the external field increases, the internal currents in the superconductor will also increase to counterbalance the additional field.

The drifting momentum is part of the electron energy in the electron tunnel. The energy required to raise the electrons in superconductors is supplied from the environment, as demonstrated by equation (13). When an electron returns to an electron hole, the stored energy is released back into the environment, ensuring that the total energy of the system remains conserved. In conductors, electrons are elevated to create currents by the external energy, which dissipates as heat when the electrons drop back to lower orbitals. Consequently, the currents will eventually cease without the continuous supply of external energy.

However, the BCS theory cannot adequately explain the Meissner effect. The lack of a mechanism to drive the Cooper pairs in the theory is one limitation, as motionless electrons cannot be deflected by a magnetic field because $F = 0$ when $v = 0$ and $E = 0$.

Critical Current Density

The critical current density refers to the maximum current density that a superconductor can tolerate. According to Stefan-Boltzmann's law, the radiation power, *P*, emitted by a blackbody is directly proportional to the fourth power of the absolute temperature, *T*:

$$P = pT^4 \tag{43}$$

Here, *p* is the Stefan-Boltzmann constant. This law can also be derived by integrating Planck’s law over the frequency and then over the solid angle. The current density in a body, such as a

superconductor, is related to the density of electrodrifts. Each electrodrift starts from an electron excitation as a result of energy absorption. At equilibrium, the lifting energy for currents in a superconductor is absorbed from the environment, as given by equation (13), and the rate of energy absorption is equal to the radiation from the superconductor.[14] Consequently, the current density and the rate of energy absorption by the superconductor are also proportional to the fourth power of its temperature. By introducing a conversion efficiency, *C*, the current density can be expressed as:

$$J = CT^4 \quad (44)$$

Here, *C* is a property specific to the superconductor. This equation indicates that the maximum current density of a superconductor is proportional to and limited by the fourth power of the temperature, meaning that the critical temperature determines the critical current density of a superconductor. The proposed model further predicts that critical temperature increases with pressure, and so does critical current density. This explains why conventional low-temperature superconductors typically have lower critical current densities than high-temperature superconductors obtained at high pressures.

Critical Magnetic Field

When subjected to an external magnetic field, a superconductor will lose its superconductivity once the applied field exceeds a certain intensity, known as the critical magnetic field. At low intensities, an external magnetic field is expelled from a superconductor in the Meissner effect. As the intensity of the external field increases, the internal field will increase proportionally to compensate for the applied field.

The compensation field arises due to the presence of internal currents, and its intensity is directly proportional to the density of the internal currents. As discussed previously, there is a limit to the maximum current density in a superconductor for a given temperature. Beyond the critical current density, the external field is no longer fully compensated inside the superconductor, and the applied field cannot be entirely expelled by the Meissner effect.

Once the critical current is exceeded, the Lorentz effect kicks in. In the remaining external field, the moving charges, including the orbital electrons, in the superconductor are deflected by the Lorentz force. The electron orbital orientations surrounding each atom are altered in a way that the electron clouds are deformed and compressed along the direction of the applied field. Consequently, the electron clouds are flattened perpendicular to the direction of the applied field, as illustrated in Figure 9B. Valence electrons play a significant role in superconductivity and are affected the most due to the less tangled effect from other electrons in the same atom. When the applied field is strong enough, i.e., over the critical field, the valence electrons will be dislocated from the electron tunnel, leading to the destruction of superconductivity.

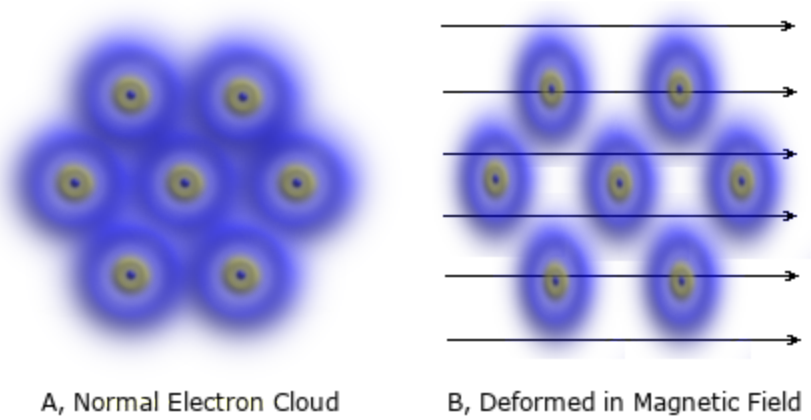

Figure 9. Superconductivity is destroyed in a magnetic field. (A) In the absence of or in a weak external magnetic field, the applied field can be completely compensated by the internal field due to the Meissner effect, as if there is no external field present. The electron cloud of valence electrons extends normally in all directions. Superconductivity may withstand a magnetic field up to a critical intensity. (B) However, above the critical intensity, an external field cannot be completely offset due to the limit of the critical current. Electrons are deflected by the Lorentz force in the remaining field and divert their orbiting plane in the direction perpendicular to the external field. This action compresses the shape of the electron clouds like squashed lanterns. The deformation of the orbital causes the valence electrons to drop below the electron tunnels, leading to the destruction of superconductivity.

The failure of superconductivity above the critical magnetic field is due to the limit of the critical current density. As predicted by equation (44), a higher temperature corresponds to a higher critical current density. Higher critical current density, in turn, sustains a higher critical magnetic field. This explains the strong correlation observed between critical temperatures and critical magnetic fields.

Type-II Superconductor

A type-II superconductor has two critical fields: B_{c1} and B_{c2}.[25-27] Normal superconductivity is observable in an applied magnetic field below B_{c1}. Above B_{c2}, the superconductivity is destroyed. Between the two fields, the superconductivity is partially destroyed in certain areas of the superconductor. These non-superconductive islands are known as magnetic vortices. The density of the vortex increases with the intensity of the applied magnetic field. Type-II superconductors are usually made of alloys or compounds.

Figure 10 illustrates a crystal structure of a superconductor composed of two distinct types of molecules. Normal superconductivity is observable in a magnetic field $B < B_{c1}$, as shown in

Figure 10A. This occurs because the applied field is below the critical field for all molecules, and every molecule is superconducting. Complete superconductivity is destroyed in a field $B > B_{c2}$, as illustrated in Figure 10C. B_{c2} represents the maximum critical field for all materials, and when the field exceeds B_{c2}, superconductivity is destroyed for every type of material.

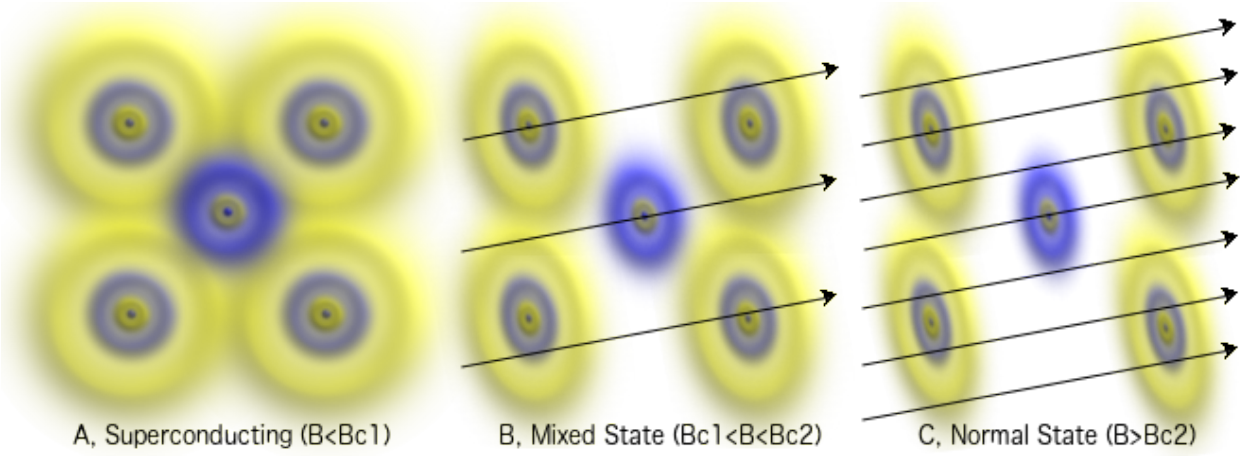

Figure 10. The responses of a type-II superconductor to different intensities of external magnetic fields are illustrated using a superconductor composed of alloys. (A) In a magnetic field below the minimum critical intensity, electron tunnels between all molecules are interconnected, and superconductivity is maintained entirely. (B) In a field between the minimum and maximum critical intensities, some valence electrons are dislocated below the electron tunnels, such as the valence electrons of the small molecule at the center, resulting in vortices or superconducting holes in the mixed state around the small molecules. (C) In a field above the maximum critical intensity, the entire superconductivity is destroyed as all the valence electrons are pushed out of the electron tunnels. Additionally, note that the deformation of the electron cloud is affected by the direction of the applied field, which explains why the critical fields of a type-II superconductor may vary depending on the direction of the external field.

In a field between B_{c1} and B_{c2}, as depicted in Figure 10B, the superconductivity for the central molecule is destroyed while other molecules surrounding it remain superconductive. The center part becomes a non-superconductive island, corresponding to the magnetic vortex, which allows the magnetic flux to penetrate.

The crystal structure of type-II superconductors typically exhibits different arrangements of molecules in various directions, particularly in compounds. Magnetic fields applied in different directions can cause diverse deflection/flattening effects on the electron clouds. Consequently, superconductivity may be destroyed at different field intensities depending on the direction of the applied field. This effect is also illustrated in Figure 10. Two critical magnetic fields are observed with an external field applied at a small angle. Different critical fields should be observed with a field applied at another angle. As the geometry of the deformed electron clouds

changes with the direction of the applied field, the valence electrons may be dislocated from the electron tunnels at different field intensities in various directions.

Since type-II superconductors are generally composed of alloys or compounds, each type of molecule can form one or more bonds with different adjacent molecules in various directions. There is at least one critical field for each type of molecule in a type-II superconductor. Consequently, the number of critical fields is typically greater than the number of types of molecules.

The molecular structure of compounds is often asymmetrical, which leads to various forms of bonding between molecules. These bonds are likely to be disrupted differently under varying conditions. This is another reason why there may be more critical magnetic fields for type-II superconductors than there are types of molecules.

The two critical fields frequently mentioned in literature are likely the minimum and maximum of all the critical magnetic fields present in a type-II superconductor. In the mixed state, vortices represent regions where superconductivity is partially disrupted. As the applied field increases and different critical fields are successively exceeded, the vortex density also increases. Vortices contribute to flux pinning, also known as quantum locking in quantum levitation. These properties make type-II superconductors particularly intriguing and valuable for various applications.

Superconductor, Insulator, and Density

Figure 11 displays known superconductive elements in blue boxes, which correspond well with the elements that are still not in the gaseous state at *3500 K* in Figure 12. This observation is consistent with the proposed theory, which suggests that the good candidates for superconductors are materials with a small gap between the conducting orbital radius and valence orbital radius, or more specifically small conducting orbital radius relative to the valence orbital radius based on equation (24) in the model. Highly repulsive molecules are likely to push the molecules apart and turn into gas more easily. The elements that have not transitioned to gases at *3500 K* are less repulsive and have smaller intermolecular distances and a smaller gap between the conducting orbital radius and valence orbital radius. A smaller molecular gap corresponds to less lifting energy required to create currents. So, materials composed of these molecules are promising candidates for superconductors. Ceramics have high melting points and are commonly used in rocketry as heat insulators. In the same logic, this characteristic indicates that ceramics have less intermolecular repulsion and smaller molecular distances, making them good candidates for superconductors according to the proposed theory. Therefore, it is not surprising that some ceramics, which were expected to be insulators, become superconductors at high pressures.

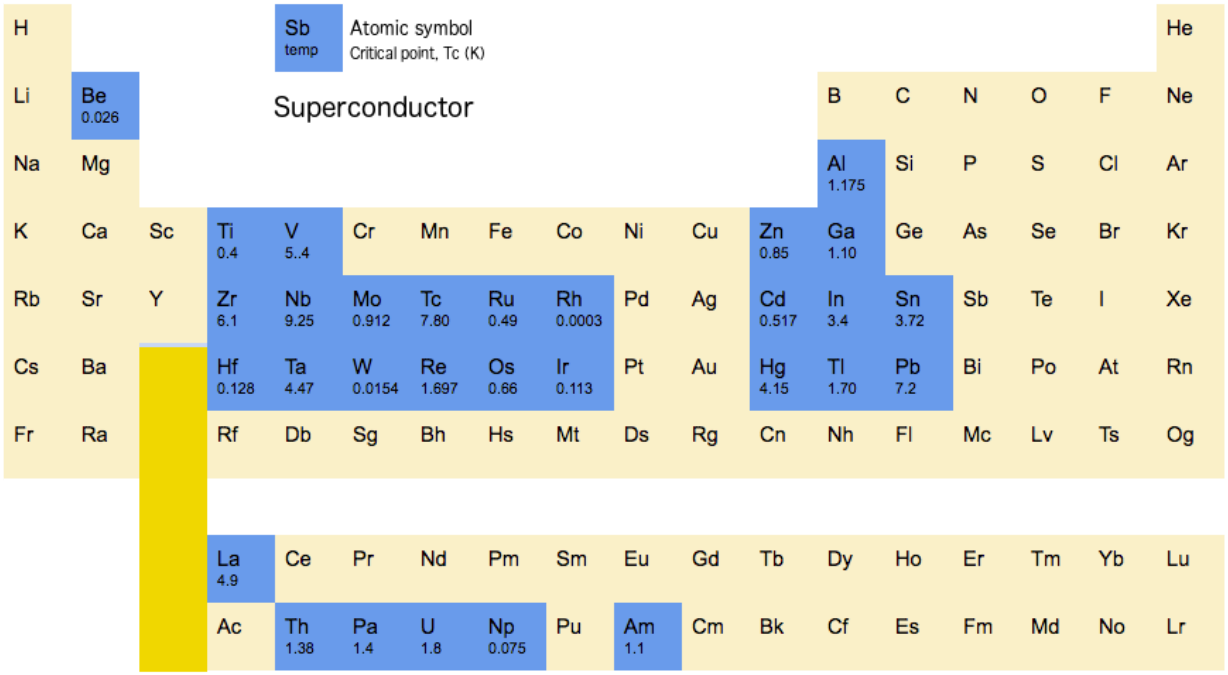

Figure 11. Known superconductive elements (T_c from Peter J. Lee's page).

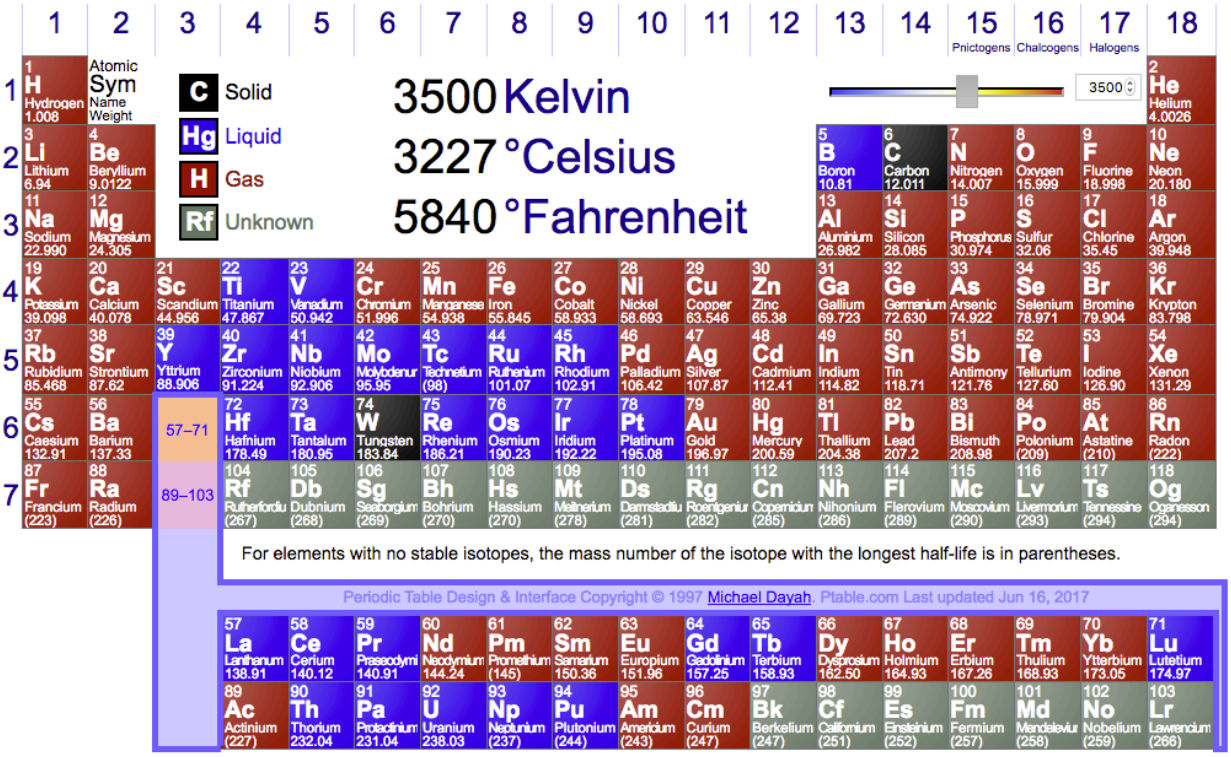

Figure 12. The states of elements at *3500 K* (screenshots from the ptable site).

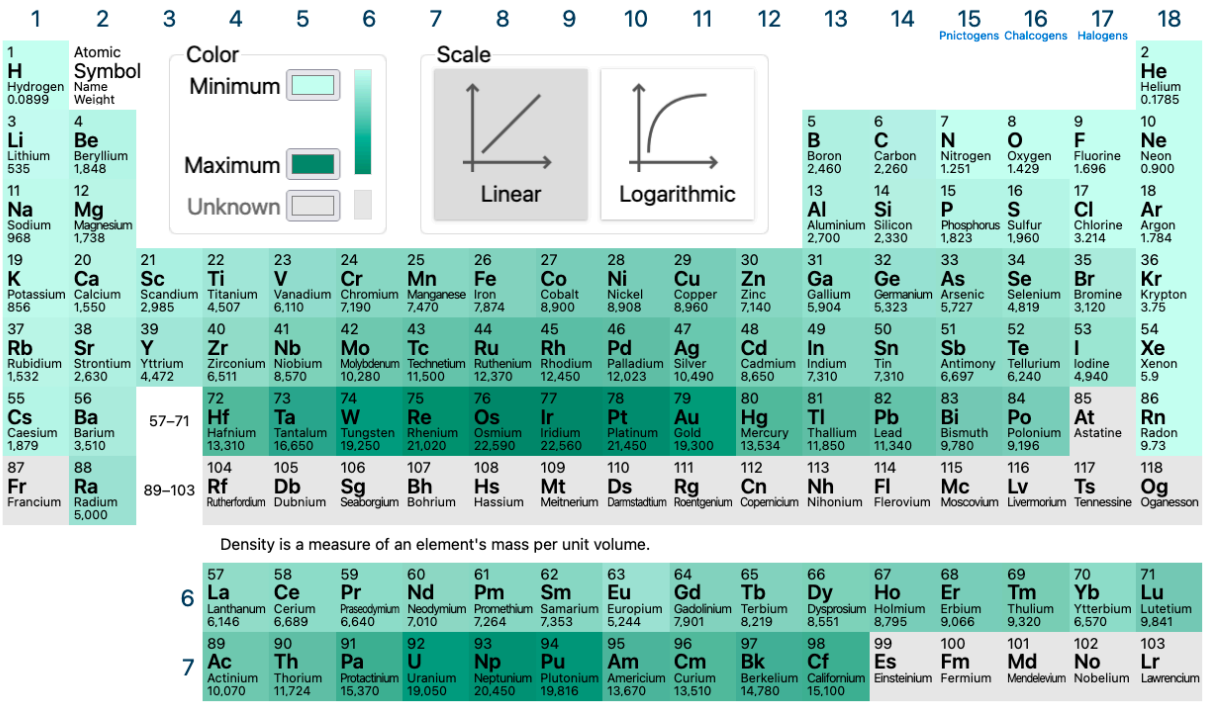

Figure 13. The densities of elements (screenshots from the ptable site).

The density of pure elemental substances is influenced by atomic numbers and the distance between molecules. If the distance between molecules is the same, one might expect materials with higher atomic numbers to have greater density. However, as illustrated in Figure 13, this trend does not hold. Instead, elements with higher density align closely with the superconductive elements presented in Figure 11. This observation implies that the distance between molecules primarily dictates the state of superconductivity. This finding also enforces the proposed theory, suggesting that a smaller molecular distance is more conducive to the formation of electron tunnels and the emergence of superconductivity.

Flux Quantization

Magnetic flux is a measurement of the total magnetic field through a given area. A looping current will result in magnetic flux. The minimum value of flux is created by a single electron circulating in an area, making magnetic flux quantized. The value of the flux quantum Φ_0 can be determined by applying some gauge transformations to the Schrödinger equation. Since the phase of the wave function depends on the gauge, but the physical predictions do not, we can conclude that the value of Φ_0 is a gauge-invariant quantity:

$$\Phi_0 = \frac{h}{2e} \tag{45}$$

Here, both *h* and *e* are fundamental physical constants for the Planck constant and the charge of an electron, respectively. This prediction may be verified using a superconductor in a donut shape.

In the proposed theory, currents in a superconductor originate from an electron's orbital jump, simultaneously creating an electron hole and an excited electron. The drifting of these charges results in one flow of negative charge and another of positive charge. The minimum flux is produced by the two flows moving in opposite directions around a donut-shaped superconductor, which is equivalent to two electrons traveling in the same direction around the donut. Consequently, the proposed theory predicts the minimum flux in a donut-shaped superconductor to be twice the flux quantum Φ_0. This prediction is confirmed experimentally by B. S. Deaver and W. M. Fairbank,[28] and independently by R. Doll and M. Näbauer.[29]

Engineering Room-Temperature Superconductors

With the understanding of the microscopic nature of superconductivity, the pursuit of room-temperature superconductors is no longer a random endeavor but becomes a deliberate engineering task. While the electron tunnel theory predicts superconducting to be an ordinary state of matter, the vast majority of substances exhibit superconductivity only under extremely high pressures and/or low temperatures. A primary challenge in attaining superconductivity is the repulsion between molecules, which intensifies as the temperature rises, particularly for symmetrical, single-atom molecules. For practical applications, it is necessary to find or develop superconductors that can operate under normal conditions on Earth. To achieve this goal, the engineering task needs to leverage intermolecular attractions to overcome the repulsions. By arranging molecules to introduce attractions between certain molecules, these molecules can be compressed nearby, and it becomes possible to create expansive electron tunnels that overlap with valence orbitals.

Electronegativity[30] plays a crucial role in the selection of elements for engineering superconductors. It is important to avoid elements with excessively high electronegativity, as they tend to tightly hold onto electrons, hindering the flow of current between molecules. Conversely, elements with insufficient electronegativity are unable to establish the necessary intermolecular attractions needed to develop wider electron tunnels. To prevent certain atoms from excessively retaining electrons, it is advantageous to choose elements with a narrow range of electronegativities. By maintaining a close range of electronegativities among the selected elements, it becomes possible to strike a balance that promotes the formation of interconnected electron tunnels and facilitates optimal electron flow between different molecules.

The connectivity of the electron tunnel between molecules is also essential for superconductivity. The molecular structure of compounds and alloys that incorporate a combination of large and small atoms gives rise to irregular intermolecular fields and forces, thereby increasing the likelihood of fostering intermolecular attractions. However, substances

composed of large and complex compounds may easily introduce electron tunnel gaps between molecules. So, it is essential to steer clear of excessively complex, large compound molecules as they can disrupt the connectivity of electron tunnels, similar to the situation observed in most insulators.

Summary

This study addresses fundamental misconceptions about electrical current and resistance, which hinder progress in the field of superconductivity. By introducing a new concept of the electron tunnel based on a mathematical model, an alternative theory is proposed for superconductivity. The theory also predicts that materials can exhibit various electrical resistance states—including superconducting, conducting, and insulating states—at different pressures and temperatures. Superconductivity is considered an ordinary state of matter, although it is not commonly observed under Earth's conditions. The theory unifies these distinct electrical resistance states through the concept of the electron tunnel, which typically develops through attraction, like compression bonding, between molecules nearby.

The model provides a deeper understanding of superconductivity and offers a comprehensive explanation of the properties and phenomena observed in superconductors. Achieving superconductivity requires positioning molecules very close together despite the repulsion between them. To date, most superconductors have been discovered through either low temperatures or high pressures, both of which are unfavorable conditions on Earth.

To achieve superconductivity under normal Earth conditions, we must find or design molecules by organizing various atoms in a manner that allows attractive forces to develop between them. These attractions should be strong enough to overcome repulsions and establish a continuous electron tunnel between molecules, broad enough to overlap the valence orbitals. This approach may lead to the discovery of superconductors with practical applications at more accessible pressures and temperatures.

Acknowledgments

We would like to thank Peter J. Lee for providing the critical temperatures for superconducting elements in the periodic table. Credit is also given to ptable.com for the screenshots of the melting points of elements from their web pages. Additionally, we acknowledge Albert Y. Liu and Thomas S. Zhou for their helpful discussions. This work has been supported by Ling Zhang.

See Also

- Unified Theory of Low and High-Temperature Superconductivity (PDF)
- Superfluids Are Not Fluids (PDF)
- Electron Tunnel (PDF)
- The Cause of Brownian Motion (PDF)
- The Process Driving Crookes Radiometers (PDF)
- Misconceptions In Thermodynamics (PDF)
- Superconductor Origin of Earth's Magnetic Field (PDF)
- Tidal Energy Is Not Renewable (PDF)
- How to Understand Relativity (PDF)
- The Simplest Derivation of $E = mc^2$ (PDF)

Revision History

- 06/02/2019: Unified Theory of Low and High-Temperature Superconductivity
- 01/16/2023: Introducing Concepts of Electron Tunnel and Compression Bond

References

1. Onnes, H.K. (1911). "*The resistance of pure mercury at helium temperatures*". Commun. Phys. Lab. Univ. Leiden. **12**: 120.
2. Van Delft, D. & Peter Kes, P. (2010). "*The Discovery of Superconductivity*". Physics Today. **63** (9): 38–43. doi:10.1063/1.3490499.
3. Bardeen, J.; et al. (1957). "*Theory of Superconductivity*". Physical Review. **108**. p. 1175. doi:10.1103/physrev.108.1175.
4. Bednorz, J.G. & Müller, K.A. (1986). "*Possible high T_c superconductivity in the Ba−La−Cu−O system*". Z. Phys. *B*. **64** (1): 189–193. doi:10.1007/BF01303701.
5. Wu, M.K.; et al. (1987). "*Superconductivity at 93 K in a New Mixed-Phase Y-Ba-Cu-O Compound System at Ambient Pressure*". Physical Review Letters. **58** (9): 908–910. doi:10.1103/PhysRevLett.58.908. PMID 10035069.
6. Drozdov, A.; et al. (2015). "*Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system*". Nature. **525** (2–3): 73–76. arXiv:1506.08190.
7. Schilling, A.; et al. (1993). "*Superconductivity above 130 K in the Hg–Ba–Ca–Cu–O system*". Nature. **363** (6424): 56–58. doi:10.1038/363056a0.
8. Takahashi, H.; et al. (2008). "*Superconductivity at 43 K in an iron-based layered compound $LaO_{1-x}F_xFeAs$*". Nature. **453** (7193): 376–378. doi:10.1038/nature06972. PMID 18432191.
9. Hirsch, J.E. (2012). "*The origin of the Meissner effect in new and old superconductors*". Physica Scripta. **85** (3): 035704. arXiv:1201.0139. doi:10.1088/0031-8949/85/03/035704.
10. Drude, P. (1900). "*Zur Elektronentheorie der Metalle*". Annalen der Physik. **306** (3): 566–613. doi:10.1002/andp.19003060312.

11. Machado, A.; et al. (2012). "*Defect Structure Versus Superconductivity in MeB2 Compounds (Me = Refractory Metals) and One-Dimensional Superconductors*", doi:10.5772/48625.
12. Vaidya, R.; et al. (2003). "*Effect of pressure on electrical resistance of WSe2 single crystal*". Pramana -- Journal of Physics, Vol. 61, No. 1, pp. 183-186.
13. Souza, E., et al. (2006). "*Improvement of metallic joint electrical conductivity using a novel conductive paste produced from recycled residues*". Rev. Esc. Minas Vol. 59, No. 2.
14. Liu, J.Z. (2022). "*Misconceptions in thermodynamics*". Stanford University.
15. London, F. (1930), "Zur Theorie und Systematik der Molekularkräfte", *Zeitschrift für Physik*, **63** (3–4): 245, Bibcode:1930ZPhy...63..245L, doi:10.1007/BF01421741, S2CID 123122363. English translations in H. Hettema, ed. (2000), *Quantum Chemistry, Classic Scientific Papers*, Singapore: World Scientific, ISBN 981-02-2771-X, OCLC 898989103, OL 9194584M which is reviewed in Parr, Robert G. (2001), "Quantum Chemistry: Classic Scientific Papers", *Physics Today*, **54** (6): 63–64, Bibcode:2001PhT....54f..63H, doi:10.1063/1.1387598.
16. Liu, J.Z. (2021). "*Superfluids are not Fluids*". Stanford University.
17. Pinceaux, J. P., Maury, J. P., Besson, J. M., Solidification of helium, at room temperature under high pressure. *Journal de Physique Lettres*. **40** (13), 307–308 (1979). doi:10.1051/jphyslet:019790040013030700.
18. Ashcroft, N. W., The hydrogen liquids. *Journal of Physics: Condensed Matter*. **12** (8A), A129–A137 (2000). Bibcode:2000JPCM...12..129A. doi:10.1088/0953-8984/12/8A/314.
19. Bonev, S. A., et al. A quantum fluid of metallic hydrogen suggested by first-principles calculations. *Nature*. **431** (7009), 669–672 (2004). arXiv:cond-mat/0410425. Bibcode:2004Natur.431..669B.
20. Souza, E., et al. (2006). "*Improvement of metallic joint electrical conductivity using a novel conductive paste produced from recycled residues*". Rev. Esc. Minas Vol. 59, No. 2.
21. Faraday, M. & Day, P. (1999). "*The philosopher's tree: a selection of Michael Faraday's writings*". CRC Press. p. 71.
22. Huray, P.G. (2010). "*Maxwell's Equations*". Wiley-IEEE. p. 22.
23. Lorentz, H.A. (1895). "*Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern*".
24. Chow, T.L. (2006). "*Electromagnetic theory*". Sudbury, MA: Jones and Bartlett. p. 395.
25. Hove, J.; et al. (2002). "*Vortex interactions and thermally induced crossover from type-I to type-II superconductivity*". Physical Review B. **66** (6): 064524. arXiv:cond-mat/0202215.
26. Callaway, D.J.E. (1990). "*On the remarkable structure of the superconducting intermediate state*". Nuclear Physics B. 344 (3): 627–645. doi:10.1016/0550-3213(90)90672-Z.
27. Abrikosov, A.A. (2003). "*Type II Superconductors and the Vortex Lattice*". Nobel Lecture.
28. Deaver, B.S. & Fairbank, W.M. (1961). "*Experimental Evidence for Quantized Flux in Superconducting Cylinders*". *Physical Review Letters*. **7** (2): 43–46.
29. Doll, R. & Näbauer, M. (1961). "*Experimental Proof of Magnetic Flux Quantization in a Superconducting Ring*". *Physical Review Letters*. **7** (2): 51–52.

30. Jensen, W.B. (1996). "*Electronegativity from Avogadro to Pauling: Part 1: Origins of the Electronegativity Concept*". *Journal of Chemical Education*. 73 (1): 11-20. Bibcode:1996JChEd..73…11J. doi:10.1021/ed073p11.